

# Competitive Markets and Boundedly Rational Expectations\*

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## Abstract

We analyze the trade of differentiated products when firms set both base and potentially hidden add-on prices. Boundedly rational consumers are subject to coarse reasoning and mistakenly believe that their action (product choice or substitution effort) has no effect on the probability of paying add-on prices. However, all consumers correctly anticipate their equilibrium expenses, so there are no “surprise charges.” Shrouding equilibria with inefficient trade exist in this setting, but only if the market is sufficiently competitive. There is an optimal maximal add-on price that defines the scope for profitable exploitative innovation. Further, the presence of boundedly rational consumers can generate innovation incentives that improve welfare relative to the rational consumer benchmark. For credit/debit card markets the model explains why informational interventions often have only small effects on behavior, while add-on price regulation increases consumer surplus.

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# 1 Introduction

Many products are offered with complex pricing schedules that comprise both base prices and add-on charges for additional services. Classic examples are credit or debit cards, mobile-phone services, and actively managed mutual funds. The add-on charges are often large relative to the base prices. In some markets, firms earn substantial economic profits from selling additional services at high add-on prices even when the industry is fairly competitive. From an industrial organization perspective, this phenomenon is not easy to rationalize as competition should drive down the sum of base and add-on prices and hence firms' profits.

To provide an explanation, Verboven (1999) and Ellison (2005) consider settings where firms can advertise base, but not add-on prices. An equilibrium then exists where firms earn economic profits from charging high add-on prices despite competition. However, the assumption of prices that cannot be advertised is difficult to justify as it should be simple for firms to make their pricing schedules transparent to consumers. Therefore, Gabaix and Laibson (2006) propose that some consumers are "myopic" (or "naive") and do not take add-on prices into account as long as they are not explicitly advertised. They show that competition between firms is then not enough to make prices transparent as educating myopic consumers about add-on charges would make them unprofitable for firms. Heidhues et al. (2017) expand on this mechanism and show that the presence of myopic consumers may induce firms to sell "deceptive" inferior products, which would reduce welfare. Hence, high add-on charges may imply a case for public policy to educate consumers or to impose disclosure requirements.

The myopic consumer argument is, however, not entirely convincing in some markets. Credit and debit cards are "high-frequency products" (Gathergood et al. 2021) that provide quick feedback to consumers. Credit card fees are made salient on credit card statements. Overcharge fees for debit card transactions are also communicated quickly to consumers. Since consumers have to manage their finances continuously, it does not seem plausible that a large fraction of them remains unaware about interest charges and penalty fees for a long time. Indeed, the empirical research on consumers' use of credit and debit cards (which we describe in detail below) provides little indication that their behavior changes substantially when interest charges and penalties are highlighted through interventions.

In this paper, we consider a market for products with complex pricing schedules in which some consumers are boundedly rational. Instead of assuming myopic beliefs, we suppose that these consumers apply coarse reasoning to their environment. They do not understand how expenses are correlated with their behavior, but they correctly anticipate the distribution over equilibrium expenses. Since all consumers have correct expectations on the equilibrium path, there are no "surprise charges." The model rationalizes high add-on prices and economic

profits in competitive settings, without the assumption of non-advertisable prices and with equilibrium beliefs that are plausible for frequent firm-consumer interactions. It generates new results on the role of competition and regulation in markets with complex pricing schedules. Moreover, it provides a natural prediction for add-on prices, and it offers a new – slightly more positive – perspective on innovation incentives in such markets. Finally, the model reconciles empirical findings in credit/debit card markets as well as in the market for mutual funds.

To discipline our approach, we directly build on the multi-product version of Heidhues et al. (2017, henceforth HKM17), and, at a later stage, also on Gabaix and Laibson (2006, henceforth GL06). As in these models, firms set base and add-on prices, and can choose whether to educate consumers by advertising add-on prices. There are rational and boundedly rational consumers. The latter ones become rational when at least one firm advertises add-on prices. To allow for competition effects, we assume that products are horizontally differentiated. Importantly, the only significant change we make relative to HKM17 and GL06 is the consumers' belief formation process. All consumers observe or correctly anticipate firms' base and add-on prices. A consumer's action – product choice in the HKM17 setting and substitution effort in the GL06 framework – affects her probability of paying an add-on charge. Rational consumers understand this link. Boundedly rational consumers mistakenly believe that their action has no influence on the probability of paying the add-on charge. For example, both passive and actively managed funds are just “investment funds” for these consumers so that they do not take into account that the former product type is less costly for them. When boundedly rational consumers select an action in equilibrium that frequently triggers the billing of additional charges, they extrapolate that this is also the case for all alternative actions. Thus, they may undervalue products where add-ons are charged infrequently or underestimate the benefits of substitution effort. One may suspect that this change in the belief formation process only rescales equilibrium outcomes. However, we show that it generates several new results.

*Shrouding equilibria and competition.* We first study under what circumstances a shrouding equilibrium with inefficient trade exists in our framework. Inefficient trade means that boundedly rational consumers purchase an inferior product (in the HKM17 setting) or that rational consumers exert substitution effort to avoid add-on charges (in the GL06 setting). We find that shrouding equilibria with inefficient trade exist in our framework, but only if the market is sufficiently competitive. As the market becomes less competitive, firms reap a larger share of the gains from trade. This creates incentives to maximize these gains, and hence to avoid the inefficiencies that occur when add-on prices are shrouded. In our HKM17 setting, this implies that firms only sell the superior product if competition is sufficiently relaxed.<sup>1</sup> A (local)

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<sup>1</sup>We show in an extension that this also holds if firms cannot educate consumers by advertising add-on prices. The assumption of free (and effective) consumer education is therefore not essential.

monopolist who reaps most gains from trade from the superior product would only reduce its profit if it sells the inferior product to some consumers. Similarly, in our GL06 setting, firms advertise their add-on prices if they enjoy sufficient market power. They do this to prevent rational consumers from exerting substitution effort, which would reduce the gains from trade.

Crucially, these findings rely on the fact that all consumers correctly anticipate their equilibrium expenses. If boundedly rational consumers would ignore add-on prices and add-on prices were large enough, then a monopolist would happily exploit them to earn profits beyond the monopoly level. Overall, we find that the main results in HKM17 and GL06 do not depend on differences between consumers' expectations and actual equilibrium outcomes. They can be consistent with high-frequency products and robust to learning. However, if consumers correctly anticipate their equilibrium expenses, the shrouding equilibria in these settings only obtain if the competition between firms is sufficiently intense.

*Endogenous maximal add-on price.* The model generates a natural prediction for the maximal add-on price. In a myopic consumer model, this variable is exogenously given.<sup>2</sup> Since myopic consumers do not take add-on prices into account, there is no bound on these prices that follows from the primitives of the model. In our framework, if the maximal add-on price is too high, a shrouding equilibrium with inefficient trade would not exist since boundedly rational consumers would refuse to trade with firms to avoid exploitation. We characterize the “optimal add-on price”, i.e., the maximal add-on price that maximizes industry profits. It is well-defined if the market is sufficiently competitive, and then equals the price firms would choose in an optimal cartel if all consumers were boundedly rational.

Under the optimal add-on price, the gap in surplus between rational and boundedly rational consumers widens as the market becomes more competitive. Furthermore, if the market is sufficiently competitive, the optimal add-on price “poisons” boundedly rational consumers' beliefs in the sense that these consumers strictly prefer no trade to purchasing the superior product. To illustrate, consider a (boundedly rational) consumer who purchases an actively managed fund with management fees that almost eat up all the returns. This consumer would conclude that investing in passive products generates a negative return so that she would not invest in equity at all if the actively managed fund were not available.

*Incentives for welfare-improving innovation.* We obtain new results on firms' innovation incentives. Heidhues et al. (2016, henceforth HKM16) show that, in a shrouding equilibrium with non-appropriable innovation (firms can freely copy others' findings), firms always have incentives to invest into exploitative innovation that increases the maximal add-on price, but

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<sup>2</sup>The maximal add-on price can be interpreted as the level of consumer protection provided through regulation. Alternatively, it may be limited by consumer reactions at very large add-on price levels.

no incentives to invest into product innovation which increases the payoff from the product that myopic consumers purchase.

This changes significantly with boundedly rational equilibrium beliefs. First, incentives for exploitative innovation are bounded since firms would not push the maximal add-on price above its optimal level. Second, if firms can keep the maximal add-on price at the optimal level through continuous exploitative innovation, they directly benefit from product innovations that increase the value of the inferior product to consumers. Third, in a shrouding equilibrium, firms' profits directly depend on the production costs of the inferior product. Hence, they are willing to invest into process innovation that reduces these production costs. Fourth, both product and process innovation are not limited to the elimination of welfare losses from inefficient trade. We show that the shrouding equilibrium survives if the inferior product becomes the superior product as long as the gains from trade of the two products are not too different. Therefore, the presence of boundedly rational consumers can generate market power that eventually improves welfare relative to the rational consumer benchmark.

*Implications for regulation.* Our model yields several implications for the regulation of markets with complex pricing schedules. First, interventions that inform consumers about the add-on charges of the products that they purchase may not change behavior since consumers are already aware of them. An informational intervention is only effective if it updates consumers' understanding of how alternative products and behaviors would alter their expenses. Most likely, this is difficult to achieve through simple nudging or disclosure policies. In contrast, add-on price regulation can be effective. This measure can redistribute surplus back to boundedly rational consumers and also reduce the scope for shrouding equilibria. However, our results on innovation imply that there can be a trade-off between welfare and consumer surplus. If maximal add-on prices are limited through regulation, then, in a shrouding equilibrium, firms have no incentives to invest into non-appropriable product innovation.

*Implications for empirical findings on markets with complex pricing schedules.* The model rationalizes several recent empirical patterns in markets with complex pricing schedules. For credit/debit card markets, the model explains why informational interventions have only small effects on consumer behavior, while add-on price regulation improves consumer surplus. A number of studies analyzed the effects of informational interventions on market outcomes. These results were highly anticipated in the context of low consumer financial literacy and abusive practices in retail finance. However, the informational interventions showed only small or insignificant effects (e.g., Seira et al. 2017). In contrast, the regulation of credit card fees had substantial effects on consumer surplus (Agarwal et al. 2015). Our model offers a uniform explanation for both observations.

For the mutual fund market, the model rationalizes several empirical observations. First, we show that it captures the price effects of index fund market entry on actively managed mutual funds, as described by Sun (2021). Second, it implies that expensive, inefficient products may be traded even if consumers are aware of add-on fees. This is important since the existence of add-on fees is often made salient to consumers, either because financial advisors mention them anyway during the consultation (Mullainathan et al. 2012), or because regulation such as the updated European “Markets in Financial Instruments Directive” (MiFID II) forces firms to make all costs transparent. Third, the model shows that trading inefficient products does not necessarily conflict with firms’ reputational concerns. These are substantial in the industry since consumers need to trust their broker or advisor to make investments into risky assets (Gennaioli et al. 2015, Kostovetsky 2016). Our version of the shrouding equilibrium no longer exhibits unexpected price charges that may conflict with firms’ desire to maintain a positive long-term relationship with clients.

*Related Literature.* The paper contributes to the behavioral industrial organization literature that studies how trade between firms and consumers is affected when some consumers are myopic or naive; see, e.g., DellaVigna and Malmendier (2004), Eliaz and Spiegler (2006), Grubb (2009, 2015), Miao (2010), Armstrong and Vickers (2012), Inderst and Ottaviani (2012), Dahremöller (2013), Shulman and Geng (2013), Warren and Wood (2014), Murooka (2015), Heidhues and Kőszegi (2017), Kosfeld and Schőwer (2017), Ko and Williams (2017), Johnen (2020), and Inderst and Obradovits (2022). By replacing myopic through boundedly rational equilibrium beliefs, we avoid the conflict between consumer expectations and experiences. This implies a bound on the welfare loss through inefficient trade. Moreover, firms have an incentive to reduce (or even reverse) this welfare loss through innovation.

The link between competition and shrouding of add-on prices is new in the hidden add-on price literature. However, a similar finding has been established in the price obfuscation literature, see Spiegler (2006), Carlin (2009), Piccione and Spiegler (2012), Chioveanu and Zhou (2013), and Chioveanu (2020). The difference between models of price obfuscation and hidden add-on pricing is that, in the former type of models, firms can increase the share of boundedly rational consumers by adopting non-transparent pricing schedules. A typical finding in these models is that the degree of price obfuscation increases in the number of firms (as it is less profitable to compete for rational consumers when there are more firms). Importantly, to obtain this result, it is assumed that firms cannot charge prices above the consumers’ valuation for the product. This ensures that random purchase decisions are weakly optimal for confused consumers. Without such an assumption, maximum confusion and unlimited prices would be optimal at any degree of competition. Our approach allows to avoid such an assumption while still obtaining a link between competition and obfuscation.

To model boundedly rational beliefs, we use a personal equilibrium concept that captures coarse reasoning. It is a short-cut version of Spiegel's (2016) Bayesian network model, where the decision-maker applies misspecified causal reasoning to make sense out of the data that she gets in equilibrium. Notions of coarse or analogy-based reasoning in strategic settings have been developed in several papers, see Eyster and Rabin (2005), Jehiel (2005), Jehiel and Koessler (2008), or Esponda and Pouzo (2016). Previous applications include adverse selection (Esponda 2008), learning in games (Mengel 2012), strategic information transmission (Hagenbach and Koessler 2020), as well as incentive contracts (Schumacher and Thysen 2022); see Jehiel (2021) for a comprehensive survey. This paper is the first that applies coarse reasoning to study markets with (potentially) hidden add-on prices.

The rest of the paper is organized as follows. In Section 2, we introduce our baseline model which builds on the HKM17 framework with multiple products. In Section 3, we analyze under what circumstances there exists an equilibrium in which firms shroud add-on prices and sell the inferior product to consumers. We derive optimal maximal add-on prices and study firms' innovation incentives. In Section 4, we consider a version of our model that builds on the GL06 setting with substitution effort. In Section 5, we discuss the implications of the model for credit and debit card markets, mutual fund markets, as well as for type-based price discrimination. Section 6 concludes. All mathematical proofs are relegated to the appendix.

## 2 Model

We analyze a market with horizontal product differentiation, superior and inferior products, as well as rational and boundedly rational consumers. To this end, we combine Salop's (1979) model of product differentiation with the multi-product market model from HKM17, and we require that the boundedly rational consumers' behavior and beliefs constitute a personal equilibrium. For this, we use a short-cut version of Spiegel's (2016) personal equilibrium concept.

*Basic Framework.* There is a unit mass of consumers. They are located uniformly on a circle with a perimeter equal to 1. Each consumer wants to buy at most one unit of a good. There are  $n$  firms  $i = 1, \dots, n$  located around the circle with equal distance between them. Each firm offers two products, a  $w$ -product and a  $v$ -product. A  $w$ -product generates utility  $w$  for a consumer and has unit production costs  $c^w$ , a  $v$ -product generates utility  $v$  and has unit costs  $c^v$ . While a  $v$ -product generates more utility than a  $w$ -product,  $v > w$ , it is still inferior since it is more costly to produce, that is, we have  $w - c^w > v - c^v$ . If a consumer purchases the  $u$ -product from a firm that is at distance  $d$  to her and pays the total price  $p$ , her utility equals  $u - p - td$ .

Denote by  $p_i^u$  the total price of firm  $i$ 's  $u$ -product,  $u \in \{w, v\}$ . The total price of the  $w$ -product only consists of a base price  $p_{i,base}^w \geq 0$ . The total price of the  $v$ -product consists of a

base price  $p_{i,base}^v \geq 0$  and an add-on price  $p_{i,add}^v \geq 0$  with maximal value  $p_{add}^v > 0$ . Hence, we have  $p_i^w = p_{i,base}^w$  and  $p_i^v = p_{i,base}^v + p_{i,add}^v$ . Firm  $i$ 's profit from each product  $u$  is the mass of consumers who purchase it from firm  $i$  times the markup  $p_i^u - c^u$ .

*Shrouded Prices and Boundedly Rational Consumers.* Firms can shroud add-on prices or advertise them. If firm  $i$  advertises its add-on price, consumers observe  $p_{i,base}^w, p_{i,base}^v$ , and  $p_{i,add}^v$ . If it shrouds its add-on price, consumers only observe  $p_{i,base}^w$  and  $p_{i,base}^v$ . In equilibrium, all consumers have correct expectations about the shrouded add-on price  $p_{i,add}^v$ .

Consumers differ in their understanding of what influences the probability of paying add-on prices. Rational consumers correctly anticipate that they have to pay an add-on price only if they purchase a  $v$ -product. Boundedly rational consumers neglect the influence of product choice on the likelihood of add-on charges. To capture their coarse perception of products, we introduce the following definitions. Denote by  $a \in A = \{0, 1, 2\}$  whether a consumer chooses a  $w$ -product ( $a = 2$ ), a  $v$ -product ( $a = 1$ ), or no product at all ( $a = 0$ ). A consumer's strategy is given by  $q \in \Delta(A)$  where  $q(a)$  indicates the probability with which she chooses action  $a$ . Let  $\mu$  be the belief of a boundedly rational consumer that she has to pay an add-on price if she purchases any product. In equilibrium, this belief equals her experiences, that is

$$\mu = \frac{q(a = 1)}{q(a = 1) + q(a = 2)}. \quad (1)$$

Thus, if the consumer's strategy  $q$  is to never purchase any product, she has pessimistic beliefs<sup>3</sup> of  $\mu = 1$ . A boundedly rational consumer's expected payoff  $U^{br}(a | q)$  from action  $a$  when her equilibrium strategy is  $q$  and her belief is given by  $\mu$  equals  $U^{br}(0 | q) = 0$  and

$$U^{br}(a | q) = \max_i \left[ u(a) - p_{i,base}^{u(a)} - \mu \tilde{p}_{i,add}^v - td_i \right] \quad (2)$$

for  $a \in \{1, 2\}$ , where  $d_i$  is the consumer's distance to firm  $i$ ,  $u(a)$  is the inverse of  $a(u)$ ,  $\tilde{p}_{i,add}^v$  is the expected add-on price in case it is not advertised, and  $\tilde{p}_{i,add}^v = p_{i,add}^v$  if firm  $i$  advertises its add-on price. Since beliefs  $\mu$  depend on the equilibrium strategy  $q$ , we adopt the following personal equilibrium concept:

**Definition 1** (Personal Equilibrium). *The strategy  $q$  is a personal equilibrium for a boundedly rational consumer at given and expected prices if  $a \in \arg \max_{a'} U^{br}(a' | q)$  for all actions  $a \in A$  in the support of  $q$  and the belief  $\mu$  is derived from  $q$  as indicated in equation (1).*

We can now fully describe the model and equilibrium concept. First, firms simultaneously choose base and add-on prices as well as whether to shroud or advertise add-on prices. If at

<sup>3</sup>This feature of the model is not essential for our main results, but it simplifies the analysis substantially.

least one firm advertises its add-on price, all consumers are rational.<sup>4</sup> Otherwise, the share  $\lambda$  of consumers is boundedly rational. After observing base and advertised add-on prices, consumers choose the product-firm combination that yields them the highest expected payoff according to their beliefs. The choice of a boundedly rational consumer must be a personal equilibrium. In case of a tie between firms, each optimal firm is chosen with equal probability, and in case of a tie between  $w$ -product and  $v$ -product, consumers choose the  $w$ -product.<sup>5</sup> In equilibrium, each firm maximizes its profit given the rivals' and consumers' strategies, the rational consumers' strategies are optimal for them at given (expected) prices, and the boundedly rational consumers' strategies are personal equilibria at given (expected) prices.

This equilibrium definition rules out that firms surprise consumers with unexpected overcharges. The implicit assumption here is that such overcharges are unprofitable as they hurt the firms' reputation and reduce future sales. The model thus captures in particular product markets in which firms and consumers interact frequently. A consumer who incurs an unexpected overcharge may quickly stop using the firm's product so that the transaction is not fully executed. Moreover, she may communicate her experience to her peers so that the gains from the overcharge are small compared to the subsequent loss in revenues.

### 3 Equilibrium Trade of Superior and Inferior Products

In this section, we derive the main results of our model. In Subsection 3.1, we examine the benchmark case when there are only rational consumers or when only the inferior product is available. In Subsection 3.2, we state our main result. It describes under what circumstances a shrouding equilibrium with inefficient trade exists. In Subsection 3.3, we use this result to characterize the add-on price that maximizes industry profits, that is, the "optimal add-on price." In Subsection 3.4, we consider exploitative innovation in our framework and show that it may establish the optimal add-on price in equilibrium. In Subsection 3.5, we further use this extension to examine firms' incentives to invest into product and process innovation. In Subsection 3.6, we briefly discuss how our main result would change if it is impossible for firms to educate consumers.

#### 3.1 The Benchmark Equilibrium

We first examine the equilibrium outcome when all consumers are rational. In this case, there is no scope for trade of the inferior product. Since the gains from trading the  $v$ -product are

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<sup>4</sup>Our main results do not depend on this extreme assumption, see the discussion in Subsection 3.6.

<sup>5</sup>This rules out equilibria with mixed consumer strategies.

smaller than the gains from trading the  $w$ -product, it is not profitable for firms to sell the inferior product, regardless of the intensity of competition. Therefore, only the  $w$ -product is traded in equilibrium. The following result describes the market outcome – prices and profits – in the symmetric benchmark equilibrium.

**Proposition 1** (Benchmark Equilibrium, Salop 1979). *Suppose all consumers are rational. Then only the  $w$ -product is traded in equilibrium. The unique symmetric equilibrium outcome is as follows:*

- (i) *If  $\frac{t}{n} < \frac{2}{3}(w - c^w)$ , firms share the market equally, each firm charges  $p^w = c^w + \frac{t}{n}$  and earns total profits of  $\pi = \frac{t}{n^2}$ ; this is also the unique equilibrium outcome.*
- (ii) *If  $\frac{2}{3}(w - c^w) \leq \frac{t}{n} \leq w - c^w$ , firms share the market equally, each firm charges  $p^w = w - \frac{t}{2n}$  and earns total profits of  $\pi = \frac{w - c^w}{n} - \frac{t}{2n^2}$ .*
- (iii) *If  $w - c^w < \frac{t}{n}$ , each firm enjoys a local monopoly and serves less than  $\frac{1}{n}$ th of the market; it charges  $p^w = \frac{w + c^w}{2}$  and earns total profits of  $\pi = \frac{(w - c^w)^2}{2t}$ ; this is also the unique equilibrium outcome.*

The proof of Proposition 1 is in Appendix A.1. We will frequently refer to this result throughout the paper. Case (i) is the standard case that is discussed in the literature on product differentiation. The degree of product differentiation is small enough such that firms are competing against each other and set prices so that the marginal consumers (located in the middle between two neighboring firms) earn a positive surplus. In this domain, a firm's individual profit equals  $\frac{t}{n^2}$  and thus strictly increases in transport costs. In Case (ii), the degree of product differentiation is large enough so that, in the symmetric equilibrium, firms are no longer competing against each other. Instead, they charge prices that make the consumers located in the middle between two neighboring firms indifferent between trading (with either firm) or not trading at all. These consumers become more difficult to serve as transport costs increase. Hence, in this domain, a firm's individual profit strictly decreases in transport costs. Finally, in Case (iii), the degree of product differentiation is sufficiently large so that it no longer pays off for firms to serve all consumers. Each firm enjoys a local monopoly and the marginal consumers are indifferent between trading with the closest firm and not trading at all. Again, individual firm profits are decreasing in transport costs in this domain. Figure 1 displays the firms' individual profits in the symmetric equilibrium for the three cases.

Before we analyze the general case, we consider a further useful benchmark. Suppose that there are now both rational and boundedly rational consumers, and that firms can only offer the  $v$ -product. Note from equation (1) that boundedly rational consumers then have correct beliefs

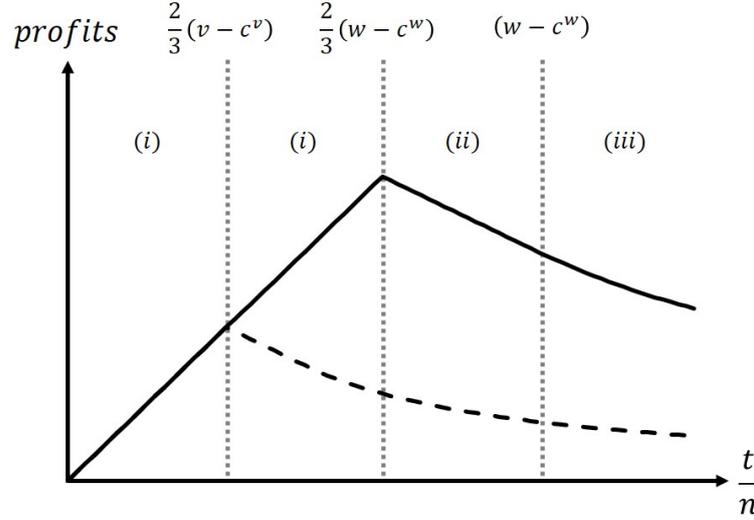


Figure 1: Individual firm profits from trading the  $w$ -product in the symmetric benchmark equilibrium (solid black line), and individual firm profits from trading the  $v$ -product in the symmetric equilibrium (dashed black line) when only the  $v$ -product is available. The numbers (i) to (iii) indicate the cases from Proposition 1.

about the probability of incurring an add-on charge after purchasing a product, regardless of their strategy  $q$ . The two consumer types then make the same choices and Proposition 1 also indicates the symmetric equilibrium outcomes for this benchmark: We only have to replace  $w - c^w$  by  $v - c^v$  in the statement. Note that, at a low degree of product differentiation,  $\frac{t}{n} \leq \frac{2}{3}(v - c^v)$ , the symmetric equilibrium profits in the two benchmark cases are identical. In this domain, competition is intense enough so that markups are determined only by transport costs, while the gains from trade,  $w - c^w$  and  $v - c^v$ , play no role for firm profits. In contrast, when the degree of product differentiation is large enough,  $\frac{t}{n} > \frac{2}{3}(v - c^v)$ , then the symmetric equilibrium profits from trading only the  $w$ -product strictly exceed those from trading only the  $v$ -product. Figure 1 illustrates this difference through the dashed line which indicates the symmetric equilibrium profits from trading the  $v$ -product.

### 3.2 The Equilibrium with Boundedly Rational Consumers

We examine the market equilibrium when some consumers are boundedly rational. Note that the outcome from the benchmark equilibrium is always an equilibrium outcome. When all firms advertise their add-on prices and act as in the benchmark equilibrium, no firm can profit from shrouding its add-on price or from charging different prices. Hence, there always exists an equilibrium in which all firms advertise their add-on prices and in which only the  $w$ -product is traded at the prices indicated in Proposition 1.

Does the presence of boundedly rational consumers create scope for the trade of inferior products? Boundedly rational consumers may not understand that the  $w$ -product offers a higher surplus to them than the  $v$ -product. Consider the following setting: All firms charge the same

prices and shroud add-on prices; they charge a positive price  $p^w$  for the  $w$ -product as well as a zero base price for the  $v$ -product; the add-on price for the  $v$ -product is  $p_{add}^v$  so that the total price for the  $v$ -product is  $p^v = p_{add}^v$ ; the total prices for the two products are such that  $w - p^w > v - p^v > 0$ .

Rational consumers anticipate the payoff from each product and purchase the  $w$ -product. In contrast, boundedly rational consumers do not correctly infer the total prices of the two products. Suppose they purchase the  $v$ -product in equilibrium. They then experience add-on charges with probability one. Since they do not take into account that the frequency of add-on charges varies in the product type, they believe that they would also have to pay the add-on price  $p_{add}^v$  for the  $w$ -product. Since the base price of the  $w$ -product is positive while the base price of the  $v$ -product is zero, we have for each firm  $i$  that

$$v - p_{i,base}^v - p_{add}^v > w - p_{i,base}^w - p_{add}^v. \quad (3)$$

Hence, the  $v$ -product appears to boundedly rational consumers as more attractive than the  $w$ -product. For these consumers, expected add-on charges are identical for both products, regardless of their equilibrium strategy. Purchasing the  $v$ -product is therefore the only personal equilibrium for boundedly rational consumers in the considered setting. We analyze under what circumstances there exists an equilibrium that features such an outcome.

**Proposition 2** (Shrouding Equilibria with Inefficient Trade). *Suppose there is a share of boundedly rational consumers and that the maximal add-on price  $p_{add}^v$  is small enough such that  $v - p_{add}^v \geq \frac{1}{3}(v - c^v)$ .*

- (i) *If  $\frac{t}{n} < p_{add}^v - c^v$ , there exists a symmetric shrouding equilibrium in which rational consumers purchase the  $w$ -product at price  $p^w = c^w + \frac{t}{n}$  and boundedly rational consumers purchase the  $v$ -product at price  $p^v = p_{add}^v$ .*
- (ii) *If  $p_{add}^v - c^v \leq \frac{t}{n} \leq \frac{2}{3}(v - c^v)$  and  $p_{add}^v > (w - c^w) - (v - c^v)$ , there exists a symmetric shrouding equilibrium in which rational consumers purchase the  $w$ -product at price  $p^w = c^w + \frac{t}{n}$  and boundedly rational consumers purchase the  $v$ -product at price  $p^v = c^v + \frac{t}{n}$ .*
- (iii) *If  $\frac{2}{3}(v - c^v) < \frac{t}{n} \leq \frac{2}{3}(w - c^w)$ , there exists no equilibrium in which the  $v$ -product is traded.*
- (iv) *If  $\frac{2}{3}(w - c^w) < \frac{t}{n} \leq w - c^w$ , there exists no symmetric equilibrium in which the  $v$ -product is traded. In any equilibrium that maximizes industry profits, firms only trade the  $w$ -product at price  $p^w = w - \frac{t}{2n}$ .*
- (v) *If  $w - c^w < \frac{t}{n}$ , there exists no equilibrium in which the  $v$ -product is traded.*

The proof of Proposition 2 is in Appendix A.1. The qualification  $v - p_{add}^v \geq \frac{1}{3}(v - c^v)$  is not essential for the result, but it saves us from further case distinctions. Figure 2 below illustrates Proposition 2 by showing the profits that firms earn in the symmetric (shrouding) equilibrium.

Consider Case (i) where the competition between firms is intense enough so that  $\frac{t}{n} < p_{add}^v - c^v$ . In this case, there exists a shrouding equilibrium with the features described above. Firms sell the  $w$ -product to rational consumers at the competitive price  $p^w = c^w + \frac{t}{n}$ . To boundedly rational consumers they sell the  $v$ -product at the total price  $p_{add}^v$ ; its base price is zero and the add-on price is maximal. Each group of consumers is convinced to purchase the best deal in the market, but only for rational consumers this is actually true. An individual firm's profit in the symmetric shrouding equilibrium equals

$$\pi^{sh} = \frac{1}{n} \left[ \lambda(p_{add}^v - c^v) + (1 - \lambda)\frac{t}{n} \right]. \quad (4)$$

The firms' strategies support an equilibrium due to the logic outlined in HKM17: Firms earn higher profits from boundedly rational consumers than from rational consumers (in Figure 2, the difference in profit levels is displayed by the black solid and the red solid line). It does not pay off for firms to educate consumers in order to sell them the superior  $w$ -product since this product can only be sold at relatively low markups.

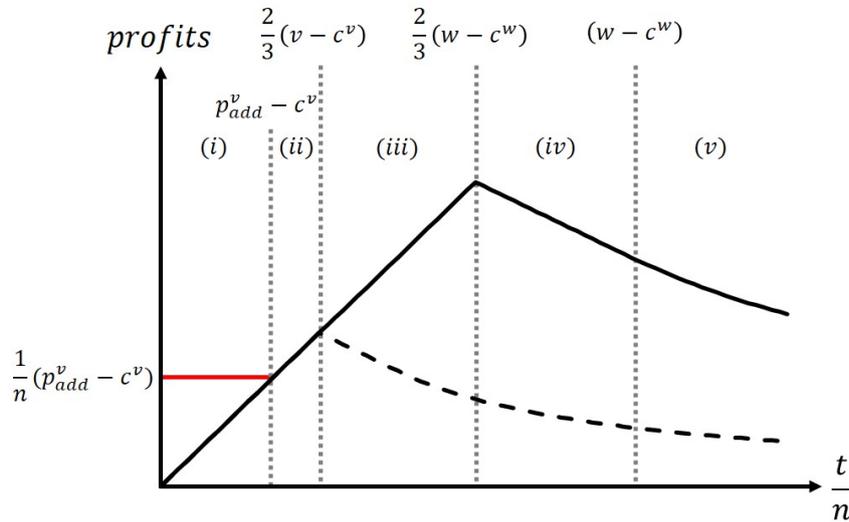


Figure 2: Individual firm profits from trading the  $v$ -product in the symmetric shrouding equilibrium (red line), individual firm profits from trading the  $w$ -product in the symmetric shrouding and in the symmetric benchmark equilibrium (solid black line), and individual firm profits from selling the  $v$ -product in a symmetric benchmark equilibrium when only the  $v$ -product is available (dashed black line). The numbers (i) to (v) indicate the cases from Proposition 2.

Next, consider Case (ii) where transport costs are such that  $p_{add}^v - c^v \leq \frac{t}{n} \leq \frac{2}{3}(v - c^v)$ . In this region, there still can exist a symmetric equilibrium in which firms sell the inferior product to boundedly rational consumers. However, they no longer benefit from this relative to the rational benchmark equilibrium outcome. The equilibrium markup on both the  $w$ - and

the  $v$ -product equals  $\frac{t}{n}$ . To charge this markup on the  $v$ -product, firms have to choose a positive base price. That is, competitive pressure is no longer enough to reduce the base price of the  $v$ -product to zero. Moreover, the maximal add-on price  $p_{add}^v$  must be large enough so that the  $v$ -product still appears as the superior deal to boundedly rational consumers. To see this, note that they strictly prefer the  $v$ -product to the  $w$ -product at the symmetric equilibrium prices if and only if

$$v - \left( c^v + \frac{t}{n} - p_{add}^v \right) - p_{add}^v > w - \left( c^w + \frac{t}{n} \right) - p_{add}^v, \quad (5)$$

which is equivalent to  $p_{add}^v > (w - c^w) - (v - c^v)$ . Overall, the only effect of trade of inferior products in this domain is that gains from trade are wasted. There is no longer a strong case for a shrouding equilibrium with inefficient trade.

Finally, consider the Cases (iii) to (v). Transport costs satisfy  $\frac{t}{n} > \frac{2}{3}(v - c^v)$  in these domains. There no longer exists a symmetric equilibrium in which the inferior product is sold to boundedly rational consumers.<sup>6</sup> The reason is that it does not pay off for firms to sacrifice gains from trade by trading the inferior product as they extract a large share of these gains. If firms would shroud add-on prices and some consumers would purchase the  $v$ -product, then at least one firm could profit from educating consumers and offering only the  $w$ -product.

The significance of Proposition 2 is that it shows when the shrouding equilibrium from the multi-product version of HKM17 also obtains in an environment where all consumers correctly predict equilibrium outcomes. Taking the Cases (i) to (v) together implies that this happens only if competition is sufficiently intense. In our version of the shrouding equilibrium, boundedly rational consumers do not learn anything on the equilibrium path that would inform them about the misspecification in their reasoning. Therefore, shrouding equilibria with inefficient trade can persist even when consumers and firms interact frequently.

Proposition 2 further shows that the scope for a shrouding equilibrium with inefficient trade depends on the gains from trade from the inferior product,  $v - c^v$ ; see the separation between Cases (ii) and (iii) in Figure 2. If these gains are small, such an equilibrium can only exist if the market is very competitive. Intuitively, this means that firms are willing to accept very inefficient trade only if markups are generally small. As markups rise, firms reap a larger share of the gains from trade, so that trading the inferior product becomes less advantageous, in particular, if  $v - c^v$  is small.

From Proposition 2 we obtain clear welfare implications. When firms strictly profit from shrouding add-on prices – in the domain of Case (i) – the surplus of boundedly rational con-

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<sup>6</sup>For the case of intermediate transport costs,  $\frac{2}{3}(w - c^w) < \frac{t}{n} \leq w - c^w$ , we cannot completely rule out that there is an asymmetric equilibrium in which at least some firms sell the  $v$ -product to boundedly rational consumers. However, such an equilibrium is not optimal for firms. In this region, the equilibrium outcome that maximizes industry profits is to only trade the  $w$ -product at the prices from the symmetric benchmark equilibrium. Thus, there is little reason to offer the  $v$ -product.

sumers is reduced by  $(v - p_{add}^v) - (w - c^w - \frac{t}{n})$ , and total welfare is reduced by  $\lambda[(w - c^w) - (v - c^v)]$  relative to the benchmark equilibrium. A policy that could increase the surplus of boundedly rational consumers is add-on price regulation, i.e., in our framework, a legislation that regulates the maximal add-on price  $p_{add}^v$  that firms can charge. Reducing the maximal add-on price also reduces the total price of the inferior product in the symmetric shrouding equilibrium. Proposition 2 implies that such a reduction has two effects. First, if the shrouding equilibrium survives the intervention, it redistributes surplus from firms back to boundedly rational consumers. Second, the intervention may render shrouding add-on prices unprofitable for firms when the new maximal add-on price  $p_{add}^v$  is small enough such that  $\frac{t}{n} \geq p_{add}^v - c^v$ . Firms then earn the same markup from both  $w$ - and  $v$ -product. Thus, keeping maximal add-on prices low may prevent the introduction and trade of inferior products.

### 3.3 Optimal Add-on Prices

When firms strictly profit from shrouding add-on prices, the total price of the  $v$ -product equals the maximal add-on price  $p_{add}^v$ . Their profit in such a shrouding equilibrium therefore depends on  $p_{add}^v$ . In the following, we characterize the maximal add-on price for the  $v$ -product that maximizes industry profits and call it the “optimal add-on price.” In the next subsection, we examine a mechanism that potentially establishes the optimal add-on price in equilibrium.

We consider first the case when competition is sufficiently relaxed so that  $\frac{t}{n} > \frac{2}{3}(v - c^v)$ . In this domain, the industry profit from selling the  $w$ -product in the benchmark equilibrium is strictly larger than the maximal industry profit from selling the  $v$ -product. This holds for any value of the maximal add-on price  $p_{add}^v$ . Hence, for maximal industry profits, the maximal add-on price is irrelevant in this domain.

Next, we assume that competition is intense enough such that  $\frac{t}{n} \leq \frac{2}{3}(v - c^v)$ . We examine which total price  $p^v$  for the  $v$ -product would firms choose in a cartel to maximize the industry profit from trading with boundedly rational consumers. This price would be chosen such that the marginal boundedly rational consumers are indifferent between trading (with either of the neighboring firms) and not trading at all, i.e.,  $v - p^v - \frac{t}{2n} = 0$ . Firms charge this total price in a symmetric shrouding equilibrium only if the maximal add-on price takes on exactly this value. As in the previous subsection, we can show that there exists a symmetric shrouding equilibrium in which firms sell the  $v$ -product at total price  $p^v$  to boundedly rational consumers, and the  $w$ -product at the symmetric benchmark equilibrium price  $c^w + \frac{t}{n}$  to rational consumers. From this, we get the following result.

**Corollary 1** (Optimal Add-On Price). *If  $\frac{t}{n} \leq \frac{2}{3}(v - c^v)$ , firms earn the maximal equilibrium industry profit in a symmetric shrouding equilibrium when the maximal add-on price for the  $v$ -product equals*

$$p_{add}^v(t, n) = v - \frac{t}{2n}.$$

*If  $\frac{t}{n} > \frac{2}{3}(v - c^v)$ , then in the equilibrium that maximizes the industry profit firms only sell the  $w$ -product to consumers so that the maximal add-on price  $p_{add}^v$  does not affect their profits.*

Figure 3 shows a firm's profit in the symmetric (shrouding) equilibrium when for each level of competition  $\frac{t}{n}$  the optimal add-on price  $p_{add}^v(t, n)$  is implemented. The red line displays the profit a firm makes from selling the  $v$ -product to boundedly rational consumers, and the black line displays a firm's profit from selling the  $w$ -product to any consumer type. Under the optimal add-on price, a firm's profit in the symmetric shrouding equilibrium equals

$$\tilde{\pi}^{sh} = \frac{1}{n} \left[ \lambda \left( v - \frac{t}{2n} - c^v \right) + (1 - \lambda) \frac{t}{n} \right]. \quad (6)$$

As the market becomes more competitive, the profits from selling the inferior product increase, while the profits from selling the superior product decrease. This reflects the differential price setting for the two products: The optimal add-on price  $p_{add}^v(t, n)$  is set to extract the maximal joint profit from selling the  $v$ -product – like in a price cartel. As  $\frac{t}{n}$  decreases, consumers become more homogeneous so that the cartel price also increases. In contrast, the price for the  $w$ -product is determined by competition between firms. Therefore, the firms' markup on this product (and hence its price) becomes smaller and smaller as  $\frac{t}{n}$  decreases. At  $\frac{t}{n} = 0$  the rational consumers' surplus equals  $w - c^w$ , while boundedly rational consumers earn no surplus at all.

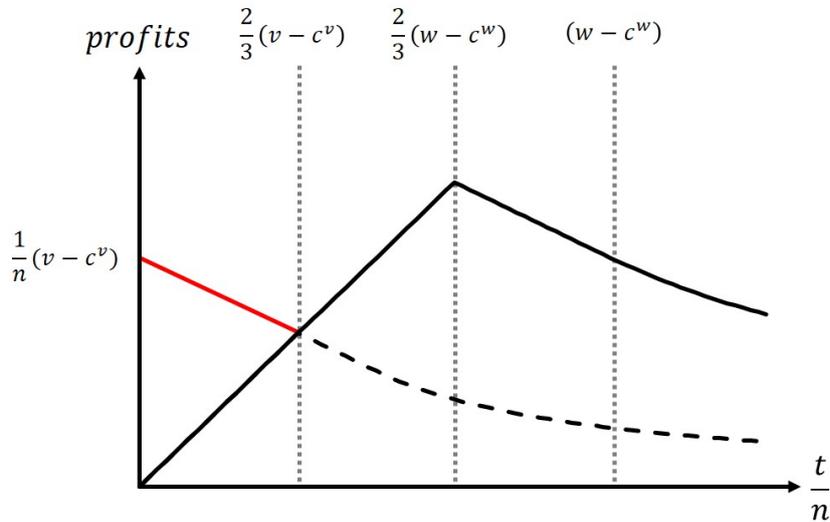


Figure 3: Individual firm profits from trading the  $v$ -product in the symmetric (shrouding) equilibrium when the optimal add-on price  $p_{add}^v(t, n)$  is implemented (red line); all other details as in Figure 2.

The symmetric shrouding equilibrium with optimal add-on price has two further features. First, almost all boundedly rational consumers would only trade with the closest firm. That is, they would prefer no trade to trade with any other firm. Second, if the market is sufficiently competitive, then boundedly rational consumers would also prefer no trade to purchasing the  $w$ -product from any firm. To see this, note that at  $\frac{t}{n} \approx 0$  we have  $w - p_{add}^v(t, n) \approx w - v < 0$ . Therefore, the extent to which boundedly rational consumers overestimate the total price of the superior product increases in the degree of competition.

### 3.4 Exploitative Innovation

Firms may be able to affect the maximal add-on price, for example, by creating new contract terms and add-on charges (e.g., new overdraft fees or extraordinary penalty payments for certain credit card transactions), or by inventing new ways to circumvent industry regulations. Such practices are often easy to copy. Once a firm establishes a new practice profitably, others can follow suit quickly. HKM16 call such developments “exploitative innovation.” In their model, myopic consumers are unaware of the new add-on charges and cannot to anticipate them. In this subsection, we adapt the HKM16 model to our framework. We essentially consider the same type of innovations. However, in our framework, all consumers take both old and new add-on charges into account; only boundedly rational consumers do not understand what they could do in order to avoid them. We show that exploitative innovation in our framework may establish the optimal add-on price as the maximal add-on price in equilibrium.

We take the model from Section 2 and add an additional stage. Before firms set their prices and choose their advertising strategies, there is an “innovation stage” where one firm, say firm 1, can invest into an exploitative innovation project. Only one project is available and after its completion, the innovation is available to all firms. We therefore focus on non-appropriable innovation. For our main application – financial services – this is the empirically relevant case for innovation projects.<sup>7</sup> Exploitative innovation increases the maximal add-on price for the  $v$ -product from  $p_{add}^v$  to  $\tilde{p}_{add}^v$ . After the innovation stage, the game continues with the realized level of the maximal add-on price. Denote by  $I_{p_{add}^v}$  the investment that firm 1 is willing to make to realize exploitative innovation, and  $\Delta p_{add}^v = \tilde{p}_{add}^v - p_{add}^v$ . We then obtain the following result.

**Corollary 2** (Exploitative Innovation). *Consider the exploitative innovation extension of our model. Suppose that  $\frac{t}{n} \leq \frac{2}{3}(v - c^v)$  and that the continuation equilibrium after the innovation stage is the symmetric shrouding equilibrium. If the maximal add-on prices  $p_{add}^v, \tilde{p}_{add}^v$  satisfy*

<sup>7</sup>In their main result (Proposition 2), HKM16 also focus on non-appropriable innovation.

$p_{add}^v < \tilde{p}_{add}^v \leq p_{add}^v(t, n)$ , firm 1 is willing to invest

$$I_{p_{add}^v} = \frac{\lambda}{n} \max \left\{ 0, \tilde{p}_{add}^v - \max \left\{ p_{add}^v, c^v + \frac{t}{n} \right\} \right\}$$

into exploitative innovation; for sufficiently small  $\frac{t}{n}$  this value equals  $I_{p_{add}^v} = \frac{\lambda}{n} \Delta p_{add}^v$ . However, if  $p_{add}^v \geq p_{add}^v(t, n)$ , we have  $I_{p_{add}^v} \leq 0$ .

When firms benefit in equilibrium from shrouding add-on prices, there are incentives for exploitative innovation as in HKM16. However, in contrast to their setting, the scope for exploitative innovation is limited in our framework due to the fact that boundedly rational consumers correctly anticipate their equilibrium expenses. Firm 1 is willing to invest into raising  $p_{add}^v$  as long as this value does not exceed the optimal add-on price  $p_{add}^v(t, n)$ . Beyond this threshold, the willingness to pay for exploitative innovation is negative since firms then would lose boundedly rational consumers and earn lower profits after the innovation. Therefore, if firms can continuously invest into exploitative innovation, the optimal add-on price  $p_{add}^v(t, n)$  would be the limit of their efforts.

### 3.5 Product and Process Innovation

An important question in industrial organization is whether firms have incentives to invest into product improvements and new technologies. Research and development are important sources of economic growth and generally benefit society. However, the trade of inferior products in a shrouding equilibrium may adversely affect firms' innovation incentives. HKM16 obtain the following result: While firms always have incentives to invest into exploitative innovation, they have no incentives to invest into product innovation which would increase the consumers' utility, provided that innovation is non-appropriable.

In this subsection, we apply the model of non-appropriable innovation from HKM16 to our multi-product framework. In contrast to HKM16, we find that, in a shrouding equilibrium with inefficient trade, firms have incentives to innovate. These incentives constitute a self-correcting force that reduces the welfare loss from inefficient trade and may even reverse the welfare implications of hidden add-on prices. To cleanly disentangle the different forces, we first consider product innovation, then process innovation, and then allow for general innovation projects that can make the  $v$ -product the superior product.

*Product Innovation.* We again assume that there is an innovation stage where firm 1 can invest into an innovation project before the market opens. This innovation is then available to all firms. Specifically, firm 1 can invest into increasing the utility of each product:  $w$ -product innovation ( $v$ -product innovation) increases the utility of the  $w$ -product ( $v$ -product) from  $w$

to  $\tilde{w} > w$  (from  $v$  to  $\tilde{v} > v$ ). Denote by  $I_w$  and  $I_v$  the investment that firm 1 is willing to make to realize the  $w$ -product and  $v$ -product innovation, respectively. Define  $\Delta v = \tilde{v} - v$ . We first assume that the  $v$ -product is the inferior product even after  $v$ -product innovation has been conducted (at a later stage, we drop this assumption). The following result provides an overview under what circumstances firm 1 would invest into product innovation.

**Corollary 3** (Product Innovation). *Consider the product innovation extension of our model. Suppose that  $\frac{t}{n} \leq \frac{2}{3}(v - c^v)$  and that the continuation equilibrium after the innovation stage is the symmetric shrouding equilibrium. If all consumers are rational, then firm 1 does not invest into product innovation,  $I_w = I_v = 0$ . If there is a share of boundedly rational consumers, then the following statements hold:*

- (i) *Firm 1 does not invest into  $w$ -product innovation,  $I_w = 0$ .*
- (ii) *Assume that the maximal add-on price  $p_{add}^v$  is given. Firm 1 then does not invest into  $v$ -product innovation,  $I_v = 0$ .*
- (iii) *Assume that maximal add-on price always equals the optimal level  $p_{add}^v(t, n)$  in the continuation equilibrium after the innovation stage. Firm 1 is then willing to invest  $I_v = \frac{\lambda}{n}\Delta v$  into  $v$ -product innovation.*

The restriction  $\frac{t}{n} \leq \frac{2}{3}(v - c^v)$  ensures that we only consider the interesting case where a symmetric shrouding equilibrium with inefficient trade can exist. First, observe that if all consumers are rational, then there are no innovation incentives in the benchmark equilibrium. In this equilibrium, firms only sell the  $w$ -product to consumers at price  $p^w = c^w + \frac{t}{n}$ , and earn profit  $\frac{t}{n^2}$ . Thus, firm profits are independent of the value of the  $w$ -product. Firm 1 has no incentive to invest into  $w$ -product innovation since any gain from innovation would be passed on to the consumers.

Next, Corollary 3 considers the case when there are boundedly rational consumers and the continuation equilibrium after the innovation stage is the symmetric shrouding equilibrium. All statements on this case directly follow from the profit functions in equation (4) and equation (6). Again, there are no incentives to invest into  $w$ -product innovation. As in the benchmark equilibrium, the markup on this product is  $\frac{t}{n}$ , regardless of its value to consumers. Similarly, firm 1 has no incentive to invest into  $v$ -product innovation as long as the maximal add-on price  $p_{add}^v$  is fixed. Since it cannot increase the price for the product, only boundedly rational consumers would benefit from an improvement of the inferior product.

This changes when the add-on price can be kept at the optimal level  $p_{add}^v(t, n)$  through continuous exploitative innovation. Firm 1 is then willing to invest into  $v$ -product innovation. Intuitively, the combination of exploitative and product-innovation means that firm 1

offers more value to boundedly rational consumers and reaps the additional gains from trade by adjusting the price in a way so that competition does not threaten the increase in profits. Statements (ii) and (iii) in Corollary 3 imply that add-on price regulation may involve a trade-off between welfare and consumer surplus. Tighter limits on add-on prices redistribute surplus from firms to boundedly rational consumers. However, they also lower firms' incentive to invest into  $v$ -product innovation, which would reduce the welfare loss from the trade of inferior products.

*Process Innovation.* We next consider the case where firm 1 can invest into an innovation project that reduces production costs. Unlike exploitative and product innovation, process innovation is not considered in HKM16, but it can be analyzed in the same fashion as product innovation. We allow for  $w$ -process and  $v$ -process innovation:  $w$ -process innovation ( $v$ -process innovation) decreases the costs of the  $w$ -product ( $v$ -product) from  $c^w$  to  $\tilde{c}^w < c^w$  (from  $c^v$  to  $\tilde{c}^v < c^v$ ). Again, we assume that the  $v$ -product is the inferior product even after  $v$ -product or  $v$ -process innovation has been conducted. Denote by  $I_{c^w}$  and  $I_{c^v}$  the amount that firm 1 is willing to invest into  $w$ -product and  $v$ -product innovation, respectively. We obtain the following result.

**Corollary 4** (Process Innovation). *Consider the process innovation extension of our model. Suppose that  $\frac{t}{n} \leq \frac{2}{3}(v - c^v)$  and that the continuation equilibrium after the innovation stage is the symmetric shrouding equilibrium. If all consumers are rational, then firm 1 does not invest into process innovation,  $I_{c^w} = I_{c^v} = 0$ . If there is a share of boundedly rational consumers, then the following statements hold:*

(i) *Firm 1 does not invest into  $w$ -process innovation,  $I_{c^w} = 0$ .*

(ii) *Firm 1 is willing to invest*

$$I_{c^v} = \frac{\lambda}{n} \max \left\{ 0, (p_{add}^v - \tilde{c}^v) - \max \left\{ p_{add}^v - c^v, \frac{t}{n} \right\} \right\}$$

*into  $v$ -process innovation; for  $\frac{t}{n}$  close enough to zero this value equals  $I_{c^v} = \frac{\lambda}{n} \Delta c^v$ .*

As for the case of product innovation, there are no incentives to invest into process innovation if a product is traded at the competitive price. Thus, there are no investments into process innovation in the benchmark equilibrium, and no investments into  $w$ -process innovation in the symmetric shrouding equilibrium. However, in the symmetric shrouding equilibrium, firms' profits decrease in the production costs of the  $v$ -product. One can see this directly from equation (4). Hence, firm 1 is willing to invest into  $v$ -process innovation. We therefore again obtain a self-correcting force that reduces the welfare loss created through inefficient trade in a shrouding equilibrium. It must be noted that – unlike the other results in this paper – the

findings on process innovation are *not* driven by the equilibrium beliefs of boundedly rational consumers. Welfare-enhancing process innovation would also occur in the HKM16 and HKM17 setting (but this is not mentioned in these papers).

*General Innovation Projects.* Product and process innovation reduce the welfare-loss from the trade of inefficient products. To what extent do innovation incentives persist if the welfare-loss is reduced to zero and the  $v$ -product becomes the superior product? In the following, we consider the general case and allow that the  $v$ -product offers more gains from trade than the  $w$ -product after innovation has taken place. We consider  $v$ -innovation that increases the value of the  $v$ -product from  $v$  to  $\tilde{v} > v$  and decreases its production costs from  $c^v$  to  $\tilde{c}^v < c^v$ . Accordingly, we define  $w$ -innovation. The maximal add-on price always equals the optimal level  $p_{add}^v(t, n)$  in the continuation equilibrium after the innovation stage. The continuation equilibrium is symmetric and equals the symmetric shrouding equilibrium whenever it exists.

We first verify that a symmetric shrouding equilibrium may exist even if the  $v$ -product becomes the superior product. In this equilibrium, all firms charge  $p^w = c^w + \frac{t}{n}$  for the  $w$ -product and  $p^v = \tilde{v} - \frac{t}{2n}$  for the  $v$ -product (with a base price of zero and the add-on price at the optimal level). We can show that, at these prices, rational consumers strictly prefer the  $w$ -product and boundedly rational consumers strictly prefer the  $v$ -product since  $\tilde{v} > w$ . Firms cannot deviate profitably without advertising add-on prices. Hence, it remains to check whether a deviation that involves advertising add-on prices can be profitable. The next result establishes that this is not the case if  $\tilde{v} - \tilde{c}^v$  is not too large relative to  $w - c^w$ .

**Proposition 3** (General Innovation Projects). *Consider the general innovation extension of our model. Suppose that  $\frac{t}{n} \leq \frac{2}{3}(v - c^v)$  and that the continuation equilibrium after the innovation stage is symmetric and equals the symmetric shrouding equilibrium whenever it exists. If all consumers are rational, then firm 1 does not invest into innovation. If there is a share of boundedly rational consumers, then the following statements hold:*

- (i) *Firm 1 does not invest into  $w$ -innovation.*
- (ii) *There is a value  $G^{max}(\lambda, t, n) > w - c^w$  so that firm 1 is willing to invest a positive amount into  $v$ -innovation if and only if  $\tilde{v} - \tilde{c}^v \leq G^{max}(\lambda, t, n)$ . The value  $G^{max}(\lambda, t, n)$  strictly increases in  $\lambda$ , and for  $\frac{t}{n} \approx 0$  we have  $G^{max}(\lambda, t, n) \approx \frac{n}{n-\lambda}(w - c^w)$ .*

We can draw two important conclusions from Proposition 3. First, the presence of boundedly rational consumers may provide innovation incentives that go beyond the elimination of welfare-losses due to inefficient trade. It can therefore lead to more welfare than in a competitive market with only rational consumers. The intuition follows Schumpeter's proposed link

between market structure and R&D investments (Schumpeter 1943, Gilbert 2006). Boundedly rational consumers create the market power necessary to make investments into innovation profitable. In particular, this holds for non-appropriable innovation. The presence of boundedly rational consumers can have the same effect as patent protection for the  $v$ -product.

Second, innovation incentives for the  $v$ -product are not unlimited. If the gains from trade from the  $v$ -product are too large relative to those from the  $w$ -product, this creates an incentive to educate all consumers and to serve a larger fraction of rational and boundedly rational consumers than in the symmetric shrouding equilibrium. This incentive increases in the number  $n$  of firms and decreases in the share  $\lambda$  of boundedly rational consumers. If the symmetric shrouding equilibrium is no longer sustainable, then in equilibrium firms charge the competitive markup  $\frac{t}{n}$ . Therefore, firm 1 refrains from innovation that destroys the symmetric shrouding equilibrium.

Overall, our results on innovation incentives provide a much more positive perspective on markets with shrouded add-on prices than the previous literature. There are incentives for exploitative innovation, but their scope is limited since consumers correctly anticipate equilibrium outcomes. Moreover, continuous exploitative innovation creates incentives for firms to increase the value of the inferior product, which reduces the welfare loss that is due to its trade. Additionally, firms have incentives to reduce the production costs of the inferior product. These innovation incentives not only reduce the welfare-loss from inefficient trade, but may even lead to a better market outcome than if all consumers were rational.

### 3.6 The Model without Education

In our baseline model, we assumed that firms can educate consumers free of charge by advertising add-on prices. This assumption is made in the hidden add-on price literature to demonstrate the robustness of shrouding equilibria, that is, to show that they can exist in a competitive setting even if education is costless. Our main result in Proposition 2 shows when shrouding equilibria with inefficient trade exist and when this is not the case. In the proofs of the statements (iii) to (v) of Proposition 2, we explicitly use the assumption of free consumer education. However, this is an extreme assumption that is most likely violated in applications. As we discuss extensively in Section 5, providing information on add-on prices often has very little impact on behavior. It therefore may be very costly to change consumers' understanding of complex pricing schedules. This raises the question to what extent our results also would obtain when it is impossible or prohibitively costly to educate consumers.

To answer this question, we consider in Appendix A.2 the model from Section 2, but assume that firms cannot advertise add-on prices to educate consumers. Firms only set prices and the behavior of boundedly rational consumers must be a personal equilibrium according

to Definition 1. We analyze the set of equilibria in this new game and obtain the following results: If the market is sufficiently competitive,  $\frac{t}{n} < v - p_{add}^v$ , then in the unique symmetric equilibrium firms sell the  $w$ -product to rational consumers at price  $p^w = c^w + \frac{t}{n}$ , and the  $v$ -product to boundedly rational consumers at price  $p^v = p_{add}^v$  (as in the symmetric shrouding equilibrium of Proposition 2). If competition is sufficiently relaxed,  $\sqrt{\frac{1}{2}}(v - c^v) \leq \frac{t}{n}$ , then in a profit-maximizing equilibrium firms only sell the  $w$ -product to consumers (other equilibria may exist though). For intermediate levels of competition, there exists a unique symmetric equilibrium in which both products are sold to consumers.

Overall, the differences between the model with free consumer education and the model without education are modest. In both models, the following holds: There is a symmetric shrouding equilibrium with inefficient trade where firms strictly benefit from shrouding if the market is competitive enough. However, if competition is sufficiently relaxed, then in the profit-maximizing equilibrium firms only trade the superior product. This symmetry between the different models arises due to a crucial feature of the basic setting: Firms can withdraw the inferior product from the market (e.g., by charging very high base prices for this product) if they do not want that its use biases the beliefs of boundedly rational consumers. This is an important difference between the multi-product version of HKM17 and other models of the hidden add-on pricing literature (such as GL06).

## 4 Shrouded Add-on Prices and Substitution Effort

The canonical model of hidden add-on pricing with myopic consumers is GL06. Their framework has been used and extended in several papers.<sup>8</sup> We therefore devote a short chapter to it. We first replicate the original result from GL06 with boundedly rational consumers who are subject to coarse reasoning, but correctly anticipate their equilibrium expenses. The main result in GL06 therefore does not depend on the assumption of consumer myopia. Next, we show that a shrouding equilibrium again only exists if the market is sufficiently competitive. This somewhat contrasts with the original version of the result, and we will explain in detail what causes this difference. All proofs for this section can be found in Appendix A.3.

### 4.1 Setup

We adapt the GL06 model to our framework. Firms offer only the  $v$ -product and the unit cost of this product equals  $c$ . Each firm  $i$  charges a base price  $p_{i,base} \in \mathbb{R}$  and an add-on price  $p_{i,add} \geq 0$

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<sup>8</sup>In particular, Armstrong and Vickers (2012), Dahrenmüller (2013), Heidhues and Köszegi (2017), Ko and Williams (2017), Kosfeld and Schüwer (2017), and Johnen (2020) directly build on the GL06 framework.

with maximal value  $p_{add} > 0$  for the product. It can advertise its add-on price or not. If firm  $i$  advertises its add-on price, all consumers observe both  $p_{i,base}$  and  $p_{i,add}$ . Otherwise, they only observe  $p_{i,base}$  and, in equilibrium, correctly anticipate  $p_{i,add}$ . Consumers who purchase the  $v$ -product can choose whether to exert substitution effort in order to avoid the add-on charge. This effort creates personal costs of  $e > 0$  for the consumer.

To model boundedly rational equilibrium beliefs, we proceed as in Section Section 2. Denote by  $a \in A = \{0, 1, 2\}$  the consumer's action:  $a = 2$  represents purchasing a  $v$ -product and exerting effort,  $a = 1$  purchasing a  $v$ -product and exerting no effort, and  $a = 0$  represents purchasing no product and exerting no effort. Let  $q \in \Delta(A)$  be a consumer's strategy. Boundedly rational consumers do not understand that exerting substitution effort reduces the probability of paying the add-on price. The belief  $\mu$  of a boundedly rational consumer about the probability of add-on charges after purchasing the  $v$ -product is again given by equation (1). Note that, given these beliefs, boundedly rational consumers see no value in exerting substitution effort. The personal equilibrium concept from Definition 1 naturally extends to this new setting.

We assume that the maximal add-on price  $p_{add}$  is strictly larger than the costs of substitution effort  $e$ . As in the original model, if all firms shroud their add-on prices, boundedly rational consumers remain boundedly rational. If at least one firm advertises its add-on price, all boundedly rational consumers become rational. The rest of the model (sequence of events, equilibrium definition) is the same as in Section 2.

## 4.2 Shrouding Equilibria

To examine the existence and features of shrouding equilibria, we proceed in two steps. First, we assume that firms cannot advertise add-on prices and describe the symmetric equilibrium outcome for this case. Second, we consider the full model where firms can advertise add-on prices and analyze under what circumstances the equilibrium outcome from the first step is indeed an equilibrium outcome of the full model.

We begin with the first step. Given that firms cannot advertise add-on prices, they choose the maximal possible add-on price  $p_{add}$  in equilibrium. Rational consumers then exert substitution effort and therefore only pay the base price  $p_{i,base}$  when they trade with firm  $i$ . Boundedly rational consumers do not understand that substitution effort would help them to avoid add-on charges. Since substitution effort is costly, they are not performing it and end up paying both the base and the maximal add-on price,  $p_{i,base} + p_{add}$ , when they trade with firm  $i$ . Given this consumer behavior, the symmetric equilibrium outcome is as follows.

**Lemma 1** (Symmetric Shrouding Equilibrium Outcome). *Consider the model with substitution effort of this section. Suppose that firms cannot advertise their add-on prices, and that the maximal add-on price is small enough such that  $v - c \geq 3p_{add}$ . The unique symmetric equilibrium outcome is then as follows:*

(i) *If  $\frac{t}{n} \leq \frac{2}{3}[(v - c) - (1 - \lambda)p_{add}]$ , firms serve all consumers and charge the base price*

$$p_{base} = c - \lambda p_{add} + \frac{t}{n}.$$

(ii) *If  $\frac{2}{3}[(v - c) - (1 - \lambda)p_{add}] < \frac{t}{n} \leq \frac{2}{3+\lambda}[(1 + \lambda)(v - c) - (1 - \lambda)p_{add}]$ , firms serve all consumers, the marginal naive consumers are indifferent between trading and not trading, and firms charge the base price*

$$p_{base} = v - p_{add} - \frac{t}{n}.$$

(iii) *If  $\frac{2}{3+\lambda}[(1 + \lambda)(v - c) - (1 - \lambda)p_{add}] < \frac{t}{n} \leq \frac{2}{3+\lambda}[(1 + \lambda)(v - c) + 4\lambda p_{add} - (1 + 3\lambda)e]$ , firms serve all rational consumers, the marginal boundedly rational consumers are indifferent between trading and not trading, and firms charge the base price*

$$p_{base} = \frac{2\lambda(v + c - 2p_{add}) + (1 - \lambda)(c + \frac{t}{n})}{1 + 3\lambda}.$$

(iv) *If  $\frac{2}{3+\lambda}[(1 + \lambda)(v - c) + 4\lambda p_{add} - (1 + 3\lambda)e] < \frac{t}{n} \leq (v - c) + 2\lambda p_{add} - (1 + \lambda)e$ , firms serve all rational consumers, marginal rational and boundedly rational consumers are indifferent between trading and not trading, and firms charge the base price*

$$p_{base} = v - e - \frac{t}{n}.$$

(v) *If  $(v - c) + 2\lambda p_{add} - (1 + \lambda)e < \frac{t}{n}$ , firms serve only a share of rational and boundedly rational consumers, and they charge the base price*

$$p_{base} = \frac{v + c}{2} - \frac{2\lambda p_{add} + (1 - \lambda)e}{2}.$$

The restriction on  $p_{add}$  is not essential for Lemma 1. It only saves us from a few more case distinctions. In particular, Case (i) and Case (ii) hold for any value of  $p_{add}$ . At higher values of  $\frac{t}{n}$ , it is no longer profitable for firms to serve boundedly rational consumers if  $p_{add}$  is sufficiently large (as doing so would require to set a very low base price). However, if  $v - c \geq 3p_{add}$ , firms serve at least some boundedly rational consumers at all values of  $\frac{t}{n}$ .

Lemma 1 describes which base price firms set at varying levels of competition in the symmetric equilibrium. Firms do not serve all consumers if transport costs are sufficiently large. In Lemma 1, this happens from Case (iii) onwards. Since the expenses of boundedly rational consumers are larger than those of rational consumers, the share of boundedly rational consumers firms serve is strictly smaller than the share of rational consumers. This is different from the original result in GL06 where myopic consumers ignore the add-on price.

To what extent do shrouding equilibria survive if firms are allowed to educate consumers by unshrouding their add-on prices? In the original result, shrouding add-on prices survives competitive pressure if there are sufficiently many myopic consumers. Through unshrouding, these consumers would realize that they can get advantageous deals when they exert substitution effort. The unshrouding firm cannot profitably match these deals. Therefore, it does not pay off for firms to unshroud add-on prices. To study whether this logic applies in our framework as well, we consider the different cases in Lemma 1 and obtain the following result.

**Proposition 4** (Equilibrium in the Model with Substitution Effort). *Consider the model with substitution effort of this section.*

- (i) *If  $\frac{t}{n} \leq \frac{2}{3}[(v-c) - p_{add}]$ , there exists a symmetric shrouding equilibrium whenever  $\lambda \geq \frac{e}{p_{add}}$ .*
- (ii) *If  $\frac{t}{n} < (v-c) + p_{add} - e$ , there exists a symmetric shrouding equilibrium whenever  $\lambda$  is sufficiently large.*
- (iii) *If  $(v-c) + 2\lambda p_{add} < \frac{t}{n}$ , there exists no shrouding equilibrium.*

Proposition 4 indicates under what circumstances the original result from GL06 obtains. The first statement highlights that this is the case if competition is fierce enough. In this case, if  $\lambda \geq \frac{e}{p_{add}}$ , there is a symmetric shrouding equilibrium in which boundedly rational consumers pay both the base and maximal add-on price, rational consumers only pay the base price and exert substitution effort. The only important difference to the original result is that, in our framework, boundedly rational consumers understand that they have to pay the add-on price; they just do not understand how to avoid it.

For intermediate levels of competition – starting from Case (ii) in Lemma 1 – firms would like to charge different base prices to rational and boundedly rational consumers in a shrouding equilibrium. Since this is not possible, firms lose profits from the fact that the marginal rational and boundedly rational consumers are typically at different locations, which makes shrouding less profitable. Nevertheless, this disadvantage vanishes as the share of boundedly rational consumers  $\lambda$  approaches unity. Therefore, deterrence from unshrouding remains effective if  $(v-c) + p_{add} - e > \frac{t}{n}$  and  $\lambda$  is large enough.

Finally, if competition is sufficiently relaxed,  $\frac{t}{n} > (v - c) + 2\lambda p_{add}$ , shrouding equilibria no longer exist. At these levels, firms charge prices so that they would not lose customers to rival firms after unshrouding add-on prices. It is then profitable to unshroud add-on prices in order to sell the add-on to rational consumers who otherwise would exert substitution effort. The intuition for this effect is similar to that for the main result in Proposition 2. When firms enjoy sufficient market power, they avoid inefficiencies that reduce the gains from trade.

We briefly contrast this finding with the original result. GL06 also allow for varying degrees of market power, and they show that the symmetric shrouding equilibrium is consistent with all markup levels. However, to get this result, GL06 assume a demand function that depends only on the difference between the (perceived) surplus of a firm's product and the (perceived) surplus of the rivals' best alternative product. This demand function can be micro-founded through a random utility model (Anderson et al. 1992). The consequence of this formulation is that, at any degree of market power, firms are competing against each other. In contrast, a sufficient degree of market power implies in our framework that the marginal (rational and boundedly rational) consumers become indifferent between trading with the closest firm and not trading at all. Given that all consumers correctly anticipate equilibrium expenses, this creates pressure to maximize the gains from trade by advertising add-on prices.

## 5 Implications

We discuss several implications of our model for markets with complex pricing schedules. In Subsection 5.1, we consider the market for credit and debit cards, and we examine the impact of informational and regulatory interventions on consumer behavior in this market. In Subsection 5.2, we outline several implications for the the market for mutual funds. Finally, in Subsection 5.3, we outline what the assumption of boundedly rational equilibrium beliefs entails for models of price discrimination between behavioral consumer types.

### 5.1 Informational and Regulatory Interventions in the Market for Credit and Debit Cards

The most common form to finance purchases or to withdraw cash is to use debit or credit cards. Debit cards draw on existing deposits at a bank. Credit cards allow to borrow limited amounts from the card issuer. Some services that debit and credit cards provide can be very costly. Debit card users pay overdraft fees if they spend more than they have on their account. These overdraft fees can be significantly larger than the value of the transaction that caused the overdraft. Credit cards charge monthly interest on the consumer's outstanding balance. Since

this is an unsecured loan, interest rates on credit card debt are relatively high. Additionally, there are penalty fees like over-limit and late fees.

For consumers, the cheapest way to finance purchases (the superior product) is to use a debit card and to maintain a sufficient balance so that no overdraft fees are incurred. However, a substantial fraction of consumers chooses a financing strategy that generates significant payments for interests and penalty fees (the inferior product). Our model implies that inefficient behaviors can be stable over time and that consumers who follow an inefficient financing strategy may not alter their behavior substantially if their attention is directed to charges they pay on a regular basis. There is evidence for both predictions in the empirical literature.

Several studies examine consumers' financial behavior using administrative data. Stango and Zinman (2009) analyze a two-year data set on debit and credit card use of relatively sophisticated consumers. These consumers spend substantial amounts on interests and fees (the median consumer pays 43 USD per month). Around 60 percent of all interest rates and fees could have been avoided by using available checking balances or other credit cards. For most individuals, the monthly interest payments (and the amount that could be saved) are stable over time. In terms of fees, a share of individuals pays stable fees, but for half of the sample monthly fee payments are negatively correlated over time. Thus, some consumers try to improve financial decision making after incurring fees. Ater and Landsman (2013) show for fee-based checking accounts that a substantial share of experienced consumers does not choose the best contract. Gathergood et al. (2021) find in an administrative data set spanning two years of credit card use that a share of consumers quickly learn to avoid fees by choosing an appropriate contractual arrangement, while others regularly incur high fees.

The fee structure of credit and debit cards has frequently been cited as an exploitative business practice by regulators and academics alike. Therefore, a number of studies examined whether informational interventions improve consumer behavior. Table 1 provides a summary of the results published in economics and finance journals so far. As can be seen from the table, the effects of information on behavior (credit repayment, product choice, overdrafting) are either relatively small or not significantly different from zero. The studies on credit card usage with most observations, Agarwal et al. (2015) and Seira et al. (2017), find no significant effects. These findings are quite surprising given that positive effects were highly anticipated in the context of consumers' low financial literacy.<sup>9</sup> The most significant effects are observed for overdraft usage. Grubb et al. (2022) find that automatic enrollment into an SMS-alert program reduces the amount of incurred overdraft fees. However, the effects are largest for consumers who rarely overdraft. For heavy users of overdrafts the amount of fees reduces

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<sup>9</sup>They were also surprising since publications in academic journals often exhibit a publication bias that favors positive results (e.g., DellaVigna and Linos 2022).

only by around 18 percent. Early warnings have no significant additional effect relative to just-in-time alerts. According to Stango and Zinman (2014), those individuals who change their behavior in response to an intervention do this by reducing spending, and not by choosing a different product or selecting different contract terms or keeping a higher balance.

**TABLE 1 – Overview of Information Interventions Credit and Debit Cards**

Study (observations, sample, country, duration)	Intervention (method)	Effect on product use
<i>Credit Cards</i>		
Agarwal et al. (2015) (160,000,000 credit card users, US, 5 years)	suggestion of alternative payment strategy, through CARD act (diff-in-diff, consumer and small business credit cards)	no significant effect on repayments, share borrowers adopting strategy to pay off debt $\leq$ 36 months raises by 0.4 percentage points from base of 5.3 percent
Seira et al. (2017) (167,190 borrowers, Mexico, 10 months)	(a) salient personal interest rate (b) personalized months to pay (c) overconfidence warning (all RCTs)	no significant effect on debt/account closures no significant effect on debt/account closures tiny negative effect on debt, no effect on account closures
Medina (2021) (26,069 customers from finance platform, Brazil, 9 months)	reminders via push notifications on upcoming payments (RCT)	drop of late-payment fees by 2.6 percentage points on a base likelihood of 29.1 percent (partially offset by increased overdrafting)
<i>Debit Cards</i>		
Stango and Zinman (2014) (7,448 consumer survey participants, US, 3 years)	survey questions on overdrafts (natural experiment)	reduction of overdrafting by 3.7 percentage points on a base likelihood of 30 percent
Alan et al. (2018) (108,000 customers from one bank, Turkey, 4 months)	(a) SMS-promotion of reduced overdraft fees (RCT) (b) SMS-promotion of overdraft availability (RCT)	reduction of overdrafting by 1.2 percentage points on a base likelihood of 31 percent increase in overdrafting by 0.9 percentage points (no long-run effects in both cases)
Grubb et al. (2022) (1,250,000 customers from four banks, UK, 10 months)	various early warning and just-in-time overdraft SMS-alerts (natural experiments and RCTs)	reduction of charges by 11 to 27 percent, effect is $\leq$ 18 percent for heavy users, early warning has no sig. additional effect

Finally, our model suggests that tightening the limit on add-on prices would benefit boundedly rational consumers. An example for such a regulation is the 2009 Credit Card Accountability Responsibility and Disclosure (CARD) Act. This legislation limited, in various ways, the extent to which companies could charge over-limit and late fees. Indeed, Agarwal et al. (2015) find that it reduced the amount of penalty fees paid by consumers by 1.6 percent of the borrowing volume. The effect was particularly pronounced among “high-risk” consumers who are relatively likely to pay such fees. Also, there was no reduction in the volume of credit. These findings are inconsistent with the assumption of a competitive market with rational consumers. However, they are consistent with a competitive market in which firms profit from charging (anticipated) add-on prices and a fraction of consumers do not understand how alternative behaviors would lower their expenses.

## 5.2 Implications for the Mutual Fund Market

Mutual funds are either actively managed funds where a fund manager makes investment decisions, or index funds that passively follow some stock index. Index funds are typically less costly in terms of management fees and often outperform actively managed funds (e.g., Fama and French 2010). Therefore, index funds can be interpreted as the superior and actively managed funds as the inferior product. Our model yields several implications for the market for mutual funds. First, it captures well the price effects triggered by index fund market entry. Second, our results imply that information on management fees may have very limited effects on consumer behavior. Third, the model shows that the trade of inferior products does not necessarily conflict with firms’ reputational concerns, which are substantial in this industry. We discuss each of these topics in turn.

*Price effects of index fund entry.* Actively managed funds exist since many decades, while index funds have been introduced only gradually since the 1970s. Sun (2021) analyzes the prices of mutual funds and investment flows in the different market segments when an index fund becomes available. She exploits the staggered timing of index fund market entry in the different equity categories. Her main findings on prices and investment flows are as follows: In response to an index fund entry, the management fees of actively managed funds decrease when they are sold directly, but increase by roughly the same amount when they are sold through financial advisors. Actively managed funds significantly lose market share among investors who invest directly. Financial advisors hardly recommend any index funds.

These developments are consistent with our differentiated products model. As long as only the inferior  $v$ -product (the actively managed fund) is available, all consumers have the same beliefs about its costs and payoffs, so that it is sold at the competitive price  $p^v = c^v + \frac{t}{n}$ .

When the superior  $w$ -product (the index fund) is introduced and the market enters a profitable shrouding equilibrium, rational consumers switch to the cheaper  $w$ -product which trades at the competitive price  $p^w = c^w + \frac{t}{n} < c^v + \frac{t}{n} = p^v$ . Boundedly rational consumers stick to the  $v$ -product, but they now pay a higher price for it since  $p_{add}^v > c^v + \frac{t}{n} = p^v$ .

*Fees and transparency regulation.* The existence of management fees is most likely not a secret even to consumers with little experience. In their audit study, Mullainathan et al. (2012) find that many financial advisors mention fees in the discussion with the client, even when the “client” (the auditor) has not (yet) asked about them. Typically, advisors then downplay these fees by explaining that they are “industry standard” and that the product is worth its costs. Hence, they can inform consumers about management fees without educating them about the difference between product types. This fits the setting of the present model where boundedly rational consumers anticipate the total price of the product they are purchasing, but not the total price of alternative products.

Further, as in the market for credit/debit cards, public policy tries to increase consumer surplus in the market for mutual funds by improving transparency. Since January 2018, the European Union imposes substantial requirements on firms that engage in financial advisory through an updated “Markets in Financial Instruments Directive” (Directive 2014/65/EU, henceforth MiFID II). MiFID II implements multiple regulations regarding the business model of financial advice, compensation schemes, product governance, and transparency. With respect to fees, it forces firms to disclose all costs of a product before the consumer purchases it, and, in regular intervals, also during the contractual relationship (Article 24(4)(c) of MiFID II). Thus, even when a consumer does not diggest all material that firms must provide due to MiFID II, the existence of fees is made very salient to her.

The effect of MiFID II on consumer behavior has not been analyzed yet and there exist few empirical studies on the general impact of the regulation. An exception is Loonen (2021). He conducts a survey with 267 Dutch investment advisors. In particular, he asks about how advisors evaluate the effect of different MiFID II measures on investor protection. Cost transparency is evaluated by 26.5 percent as positive or very positive for investor protection, by 44.2 percent as neutral, and by 29.2 percent of advisors as negative or very negative. Thus, there seems to be no consensus to what extent cost transparency helps investors.

*Trust and reputational concerns.* Mutual funds are often sold indirectly through brokers who also offer investment advise to their clients. A growing literature argues that the relationship to the financial advisor is important for the client who seeks investment advice. Gennaioli et al. (2015) argue that the main role of financial advisors is not to provide information, but to act as “money doctors” who help clients making risky investments by reducing their anxiety

about taking risk. They “are trusted to do so even when their advice is costly, generic, and occasionally self-serving” (Gennaioli et al. 2015, page 92). Consumers seeking advice pay substantial fees on investment products only if they sufficiently trust their advisor.

Given that firms benefit substantially from their clients’ trust, they are interested in maintaining a good relationship with their clients. Hackethal and Inderst (2012) document for a sample of German companies that most firms measure consumers’ satisfaction with the service and the probability of recommendation of the service to others. However, they mostly focus on emotional components, and not on judgments about investment performance. They also employ “mystery shopping” to monitor the quality of the service.

Further, Kostovetsky (2016) shows that for retail investors the relationship to the fund management matters. If the ownership of a mutual fund changes, there are significant outflows of investments following the announcement date. This effect is particularly pronounced for mutual funds with high management fees. Hence, for the consumers who invest into these funds the relationship to the asset management is especially important. Choi and Robertson (2020) find that two of the most important factors that determine the share invested in equity is trust in market participants and trust in financial advisors. Overall, a reduction in trust reduces investments in equity, see Guiso et al. (2008) and Gurun et al. (2018).

Our model with boundedly rational consumers shows that there is not necessarily a conflict between reputational concerns and the trade of inferior products since consumers only incur expected charges. In contrast, if consumers were myopic, the ongoing trade of mutual funds with high add-on charges would continuously surprise consumers and produce disappointed (less wealthy than expected) customer generations. This would be difficult to reconcile with an industry that is very concerned with its reputation.

### **5.3 Consumer Data and Price Discrimination**

Firms collect increasing amounts of data on consumers, which can be used to discriminate between different consumer groups. Two papers extend the GL06 framework to study discrimination between myopic and rational consumers. Heidhues and Kőszegi (2017) consider a static setting and allow for different distortions of trade when myopic consumers are exploited. Depending on the distortion, discrimination between myopic and rational consumers can have positive or negative effects on welfare. Johnen (2020) analyzes a dynamic setting where firms learn their customers’ types, and can tailor later offers to this information. He shows that a firm’s informational advantage against its rivals translates into monopoly power and positive profits even when firms compete in Bertrand manner. An informed firm can charge a high base price to rational and a relatively low base price to myopic consumers. A rival can poach both consumer groups only with a uniform price. Hence, as long as it does not charge a relatively

low price, it only attracts the unprofitable rational consumers.

When firms and consumers interact repeatedly, it matters whether types remain constant or change over time. In the model by Johnen (2020), it is assumed that myopic consumers remain myopic. This makes type information valuable for firms. In the GL06 framework, the assumption of constant types means that myopic consumers do not learn about the existence of hidden add-on prices even though they experience unexpected add-on price charges in each period. Our version of the GL06 framework from Section 4 implies that such an assumption is not needed. The myopic consumers in the original GL06 framework and the boundedly rational consumers in the present model behave in the same way in a shrouding equilibrium as long as the maximal add-on price is not too large. Hence, it is possible to obtain the main results from Heidhues and Kőszegi (2017) and Johnen (2020) in a framework where consumers are not assumed to neglect the experience of surprise charges.

## 6 Conclusion

Consumer behavior in many markets with complex pricing schedules is difficult to reconcile with rationality and standard preferences. To study such markets, it has been assumed that a share of consumers does not take the add-on charges of certain services into account. While such an assumption is reasonable for some one-time transactions, it is less convincing when consumers use a product frequently, or when firms need to build up reputation and thus wish to avoid large discrepancies between consumer expectations and actual experiences.

We therefore studied a market with complex pricing schedules where boundedly rational consumers correctly anticipate their expenses in equilibrium, but falsely predict how these expenses would change if they make different product or effort choices. The core results from the classic papers on hidden add-on pricing obtain in this model, but only if the market is sufficiently competitive. Moreover, the model generates a natural prediction for the maximal add-on price, a variable that is usually taken as given in the related literature. We showed that firms have limited incentives to invest into exploitative innovation, and that continuous exploitative innovation creates incentives to improve products even if innovation is non-appropriable. These innovation incentives are strong enough to improve welfare in the shrouding equilibrium beyond the rational consumer benchmark level. Further, our model implies that transparency regulation is ineffective as long as it does not update the boundedly rational consumers' thinking about outcomes off the equilibrium path. In contrast, limits on maximal add-on prices are effective. Both observations are in line with empirical findings in the market for credit/debit cards and in the market for mutual funds.

A promising avenue for future empirical research on markets with complex pricing sched-

ules could be the analysis of product and process innovation. There is currently very little information on the extent to which production costs and consumer utility from products change over time in such markets when price competition is relaxed through hidden add-on prices.<sup>10</sup> Casual observation of innovation in retail finance, e.g., by fintech companies, suggests that substantial innovation is taking place that potentially raises welfare through increasing consumer access and convenience. The implication of the present model would then be that hiding add-on prices in the long run also can be beneficial for welfare as it improves innovation incentives.

## References

- AGARWAL, SUMIT, SOUPHALA CHOMSISENGPHET, NEALE MAHONEY, AND JOHANNES STROEBEL (2015): “Regulating consumer financial products: Evidence from credit cards,” *Quarterly Journal of Economics*, 130(1), 111–164.
- ALAN, SULE, MEHMET CEMALCILAR, DEAN KARLAN, AND JONATHAN ZINMAN (2018): “Unshrouding: Evidence from bank overdrafts in Turkey,” *Journal of Finance*, 73(2), 481–522.
- ANDERSON, SIMON, ANDRE DE PALMA, AND JACQUES-FRANCOIS THISSE (1992): *Discrete choice theory of product differentiation*, MIT Press, Cambridge.
- ARMSTRONG, MARK, AND JOHN VICKERS (2012): “Consumer protection and contingent charges,” *Journal of Economic Literature*, 50(2), 477–493.
- ATER, ITAI, AND VARDIT LANDSMAN (2013): “Do customers learn from experience? Evidence from retail banking,” *Management Science*, 59(9), 2019–2035.
- CARLIN, BRUCE (2009): “Strategic price complexity in retail financial markets,” *Journal of Financial Economics*, 91(3), 278–287.
- CHIOVEANU, IOANA, AND JIDONG ZHOU (2013): “Price competition with consumer confusion,” *Management Science*, 59(11), 2450–2469.
- CHIOVEANU, IOANA (2020): “A more general model of price complexity,” *International Journal of Industrial Organization*, 69, 102563.
- CHOI, JAMES, AND ADRIANA ROBERTSON (2020): “What matters to individual investors? Evidence from the horse’s mouth,” *Journal of Finance*, 75(4), 1965–2020.

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<sup>10</sup>One exception is Kulasekaran and Shaffer (2002). They study the cost efficiency of credit card companies in the 1990s and find mixed results on the extent to which companies reduce their costs over time.

- DAHREMÖLLER, CARSTEN (2013): “Unshrouding for competitive advantage,” *Journal of Economics and Management Strategy*, 22(3), 551–568.
- DELLAVIGNA, STEFANO, AND ULRIKE MALMENDIER (2004): “Contract design and self-control: Theory and evidence,” *Quarterly Journal of Economics*, 119(2), 353–402.
- DELLAVIGNA, STEFANO, AND ELIZABETH LINOS (2022): “RCTs to scale: Comprehensive evidence from two nudge units,” *Econometrica*, 90(1), 81–116.
- ELIAZ, KFIR, AND RANI SPIEGLER (2006): “Contracting with diversely naive agents,” *Review of Economic Studies*, 73(3), 689–714.
- ELLISON, GLENN (2005): “A model of add-on pricing,” *Quarterly Journal of Economics*, 120(2), 585–637.
- ESPONDA, IGNACIO (2008): “Behavioral equilibrium in economies with adverse selection,” *American Economic Review*, 98(4), 1269–1291.
- ESPONDA, IGNACIO, AND DEMIAN POUZO (2016): “Berk-Nash equilibrium: A framework for modeling agents with misspecified models,” *Econometrica*, 84(3), 1093–1130.
- EYSTER, ERIK, AND MATTHEW RABIN (2005): “Cursed equilibrium,” *Econometrica*, 73(5), 1623–1672.
- FAMA, EUGENE, AND KENNETH FRENCH (2010): “Luck versus skill in the cross-section of mutual fund returns,” *Journal of Finance*, 65(5), 1915–1947.
- GABAIX, XAVIER, AND DAVID LAIBSON (2006): “Shrouded attributes, consumer myopia, and information suppression in competitive markets,” *Quarterly Journal of Economics*, 121(2), 505–540.
- GATHERGOOD, JOHN, HIROAKI SAKAGUCHI, NEIL STEWART, AND JÖRG WEBER (2021): “How do consumers avoid penalty fees? Evidence from credit cards,” *Management Science*, 67(4), 2562–2578.
- GENNAIOLI, NICOLA, ANDREI SHLEIFER, AND ROBERT VISHNY (2015): “Money doctors,” *Journal of Finance*, 70(1), 91–114.
- GILBERT, RICHARD (2006): “Looking for Mr. Schumpeter: Where are we in the competition-innovation debate?,” *Innovation Policy and the Economy*, 6, 159–215.
- GRUBB, MICHAEL (2009): “Selling to overconfident consumers,” *American Economic Review*, 99(5), 1770–1807.

- GRUBB, MICHAEL (2015): “Consumer inattention and bill-shock regulation,” *Review of Economic Studies*, 82(1), 219–257.
- GRUBB, MICHAEL, DARRAGH KELLY, JEROEN NIEBOER, AND MATTHEW OSBORNE (2022): “Sending out an SMS: Automatic enrolment experiments for overdraft alerts,” Working Paper, Boston College.
- GUIO, LUIGI, PAOLA SAPIENZA, AND LUIGI ZINGALES (2008): “Trusting the stock market,” *Journal of Finance*, 63(6), 2557–2600.
- GURUN, UMIT, NOAH STOFFMAN, SCOTT YONKER (2018): “Trust busting: The effect of fraud on investor behavior,” *Review of Financial Studies*, 31(4), 1341–1376.
- HAGENBACH, JEANNE, AND FRÉDÉRIC KOESSLER (2020): “Cheap talk with coarse understanding,” *Games and Economic Behavior*, 124, 105–121.
- HACKETHAL, ANDREAS, AND ROMAND INDERST (2012): “Messung des Kundennutzens der Anlageberatung,” Wissenschaftliche Studie im Auftrag des Bundesministeriums für Ernährung, Landwirtschaft und Verbraucherschutz (BMELV).
- HEIDHUES, PAUL, BOTOND KŐSZEGI, AND TAKESHI MUROOKA (2016): “Exploitative innovation,” *American Economic Journal: Microeconomics*, 8(1), 1–23.
- HEIDHUES, PAUL, BOTOND KŐSZEGI, AND TAKESHI MUROOKA (2017): “Inferior products and profitable deception,” *Review of Economic Studies*, 84(1), 323–356.
- HEIDHUES, PAUL, AND BOTOND KŐSZEGI (2017): “Naïveté-based discrimination,” *Quarterly Journal of Economics*, 132(2), 1019–1054.
- INDERST, ROMAN, AND MARCO OTTAVIANI (2012): “How (not)to pay for advice: A framework for consumer financial protection,” *Journal of Financial Economics*, 105(2), 393–411.
- INDERST, ROMAN, AND MARTIN OBRADOVITS (2022): “Excessive competition on headline prices,” Working Paper, Goethe-University Frankfurt.
- JEHIEL, PHILIPPE (2005): “Analogy-based expectation equilibrium,” *Journal of Economic Theory*, 123(2), 81–104.
- JEHIEL, PHILIPPE (2021): “Analogy-based expectation equilibrium and related concepts: Theory, applications, and beyond,” Working Paper, Paris School of Economics.
- JEHIEL, PHILIPPE, AND FRÉDÉRIC KOESSLER (2008): “Revisiting games of incomplete information with analogy-based expectations,” *Games and Economic Behavior*, 62(2), 533–557.

- JOHNEN, JOHANNES (2020): “Dynamic competition in deceptive markets,” *RAND Journal of Economics*, 51(2), 375–401.
- KO, JEREMY, AND JARED WILLIAMS (2017): “The effects of regulating hidden add-on costs,” *Journal of Money, Credit and Banking*, 49(1), 39–74.
- KOSFELD, MICHAEL, AND ULRICH SCHÜWER (2017): “Add-on pricing in retail financial markets and the fallacies of consumer education,” *Review of Finance*, 21(3), 1189–1216.
- KOSTOVETSKY, LEONARD (2016): “Whom do you trust?: Investor-advisor relationships and mutual fund flows,” *Review of Financial Studies*, 29(4), 898–936.
- KULASEKARAN, SIVAKUMAR, AND SHERRILL SHAFFER (2002): “Cost efficiency among credit card banks,” *Journal of Economics and Business*, 54(6), 595–614.
- LOONEN, TOM (2021): “Dutch investment advisors’ perceptions towards the MiFID II directive,” *Journal of Financial Regulation and Compliance*, 29(2), 202–217.
- MEDINA, PAOLINA (2021): “Side effects of nudging: Evidence from a randomized intervention in the credit card market,” *Review of Financial Studies*, 34(5), 2580–2607.
- MENGEL, FRIEDERIKE (2012): “Learning across games,” *Games and Economic Behavior*, 74(2), 601–619.
- MIAO, CHUN-HUI (2010): “Consumer myopia, standardization and aftermarket monopolization,” *European Economic Review*, 54(7), 931–946.
- MULLAINATHAN, SENDHIL, MARKUS NOETH, AND ANTOINETTE SCHOAR (2012): “The market for financial advice: An audit study,” NBER Working Paper No. 17929.
- MUROOKA, TAKESHI (2015): “Deception under competitive intermediation,” Working Paper, Osaka University.
- PICCIONE, MICHELE, AND RAN SPIEGLER (2012): “Price competition under limited comparability,” *Quarterly Journal of Economics*, 127(1), 97–135.
- SALOP, STEVEN (1979): “Monopolistic competition with outside goods,” *Bell Journal of Economics*, 10(1), 141–156.
- SCHUMACHER, HEINER, AND HEIDI THYSEN (2022): “Equilibrium contracts and boundedly rational expectations,” *Theoretical Economics*, 17, 371–414.

- SCHUMPETER, JOSEPH (1943): *Capitalism, Socialism and Democracy*, Harper Perennial Modern Classics.
- SEIRA, ENRIQUE, ALAN ELIZONDO, AND EDUARDO LAGUNA-MÜGGENBURG (2017): “Are information disclosures effective? Evidence from the credit card market,” *American Economic Journal: Economic Policy*, 9(1), 277–307.
- SHULMAN, JEFFREY, AND XIANJUN GENG (2013): “Add-on pricing by asymmetric firms,” *Management Science*, 59(4), 899–917.
- SPIEGLER, RAN (2006): “Competition over agents with boundedly rational expectations,” *Theoretical Economics*, 1(2), 207–231.
- SPIEGLER, RAN (2016): “Bayesian networks and boundedly rational expectations,” *Quarterly Journal of Economics*, 131(3), 1243–1290.
- STANGO, VICTOR, AND JONATHAN ZINMAN (2009): “What do consumers really pay on their checking and credit card accounts? Explicit, implicit, and avoidable costs,” *American Economic Review Papers and Proceedings*, 99(2), 424–429.
- STANGO, VICTOR, AND JONATHAN ZINMAN (2014): “Limited and varying consumer attention: Evidence from shocks to the salience of bank overdraft fees,” *Review of Financial Studies*, 27(4), 990–1030.
- SUN, YANG (2021): “Index fund entry and financial product market competition,” *Management Science*, 67(1), 500–523.
- VERBOVEN, FRANK (1999): “Product line rivalry and market segmentation with an application to automobile optional engine pricing,” *Journal of Industrial Economics*, 47(4), 399–425.
- WARREN, PATRICK, AND DANIEL WOOD (2014): “The political economy of regulation in markets with naïve consumers,” *Journal of the European Economic Association*, 12(6), 1617–1642.

## A Appendix

### A.1 Omitted Proofs from Subsection 3.1 and Subsection 3.2

*Proof of Proposition 1.* Since the  $w$ -product offers more gains from trade than the  $v$ -product, only the  $w$ -product is traded in equilibrium when all consumers are rational. Therefore, we ignore prices for the  $v$ -product in the following. **Step 1.** We derive the symmetric equilibrium outcome when  $\frac{t}{n} < \frac{2}{3}(w - c^w)$ . This is the standard case where the consumer who is indifferent between two neighboring firms  $i$  and  $j$  at prices  $p_i^w, p_j^w$  is defined by

$$w - p_i^w - td = w - p_j^w - t\left(\frac{1}{n} - d\right). \quad (7)$$

From this we get that if all other firms charge  $p_{-i}^w$ , then demand for firm  $i$ 's  $w$ -product is  $D_i = \frac{p_{-i}^w - p_i^w}{t} + \frac{1}{n}$ , and that there is a symmetric equilibrium in which all firms charge  $p^w = c^w + \frac{t}{n}$ . Indeed, the consumer at distance  $\frac{1}{2n}$  to the two closest firms then trades at this price if  $\frac{t}{n} < \frac{2}{3}(w - c^w)$ . **Step 2.** We show that the equilibrium outcome is unique when  $\frac{t}{n} < \frac{2}{3}(w - c^w)$ . Assume by contradiction that there is an equilibrium in which some firm  $i$  charges a price  $p_i^w > c^w + \frac{t}{n}$ . Assume w.l.o.g. that firm  $i$  charges the highest price. We show that firm  $i$  can deviate profitably by lowering its price. We consider firm  $i$ 's profit  $\tilde{\pi}_i$  from trading with consumers on the interval between firm  $i$  and any given neighboring firm  $j$ . We distinguish between two cases, Case (A) and Case (B). In Case (A), the marginal consumer on this interval who trades with firm  $i$  is indifferent between trading with firm  $i$  and not trading at all. This consumer is characterized by the equality

$$w - p_i^w - td = 0 \quad (8)$$

The profit  $\tilde{\pi}_i$  decreases in  $p_i^w$  if  $p_i^w > \frac{w+c^w}{2}$ , and it is maximal only if  $p_i^w \leq \frac{w+c^w}{2}$ . If  $p_i^w \leq \frac{w+c^w}{2}$ , then, by the assumption that  $\frac{t}{n} < \frac{2}{3}(w - c^w)$ , it must be the case that the marginal consumer for firm  $i$  is located further than  $\frac{1}{2n}$  away from this firm, which contradicts the assumption that firm  $i$  charges the highest price. Thus, we must have  $p_i^w > \frac{w+c^w}{2}$ . In Case (B), the marginal consumer on the interval between firm  $i$  and firm  $j$  is indifferent between the two firms and earns strictly positive surplus. This consumer is characterized by the equality in (7). We then have

$$\tilde{\pi}_i = (p_i^w - c^w) \left( \frac{p_j^w - p_i^w}{2t} + \frac{1}{2n} \right), \quad (9)$$

and  $\frac{\partial \tilde{\pi}_i}{\partial p_i^w} < 0$  if

$$c^w + \frac{t}{n} < 2p_i^w - p_j^w, \quad (10)$$

which, by assumption, is satisfied. Combining the results from Case (A) and Case (B) shows that firm  $i$  can profitably lower its price, a contradiction. Hence, there cannot be an equilibrium in which any firm  $i$  charges a price  $p_i^w > c^w + \frac{t}{n}$ . Next, assume by contradiction that there is an equilibrium in which some firm  $i$  charges a price  $p_i^w < c^w + \frac{t}{n}$ . Assume w.l.o.g. that firm  $i$  charges the lowest price. We show that firm  $i$  can deviate profitably by increasing its price. By the arguments above, we must have  $p_j^w \leq c^w + \frac{t}{n}$  for all firms  $j$  in this equilibrium. We again consider firm  $i$ 's profit  $\tilde{\pi}_i$  from trading with consumers on the interval between firm  $i$  and any given neighboring firm  $j$ . The marginal consumer on this interval must be indifferent between trading with firm  $i$  and firm  $j$ , and earn strictly positive surplus. This consumer is defined by the equality in (7) and  $\tilde{\pi}_i$  is given by equation (9). Hence, we have  $\frac{\partial \tilde{\pi}_i}{\partial p_i^w} > 0$  if

$$c^w + \frac{t}{n} > 2p_i^w - p_j^w, \quad (11)$$

which, by assumption, is satisfied. Thus, firm  $i$  can profitably increase its price, a contradiction. This completes the proof of the statement. **Step 3.** We derive the symmetric equilibrium outcome when  $\frac{2}{3}(w - c^w) \leq \frac{t}{n} \leq w - c^w$ . Note that when all firms charge the price  $p^w = w - \frac{t}{2n}$ , the consumers at distance  $\frac{1}{2n}$  to the two closest firms are indifferent between trading or not. We show that if  $\frac{2}{3}(w - c^w) \leq \frac{t}{n}$ , it does not pay off for firm  $i$  to undercut this price by some  $\epsilon > 0$  when all other firms also charge it. If firm  $i$  makes this change, the marginal consumers for firm  $i$  are defined by

$$w - \left(w - \frac{t}{2n} - \epsilon\right) - td = w - \left(w - \frac{t}{2n}\right) - t\left(\frac{1}{n} - d\right), \quad (12)$$

demand for firm  $i$ 's  $w$ -product is  $D_i = \frac{1}{n} + \frac{\epsilon}{t}$ , and firm  $i$ 's profit equals

$$\pi_i = \left(w - \frac{t}{2n} - \epsilon - c^w\right) \left(\frac{1}{n} + \frac{\epsilon}{t}\right). \quad (13)$$

Note that  $\frac{2}{3}(w - c^w) \leq \frac{t}{n}$  implies  $\frac{\partial \pi_i}{\partial \epsilon} < 0$  at any  $\epsilon > 0$ . Similarly, we can show that if  $\frac{t}{n} \leq w - c^w$ , it does not pay off for firm  $i$  to charge a price higher than  $p^w = w - \frac{t}{2n}$ . Hence, if  $\frac{2}{3}(w - c^w) \leq \frac{t}{n} \leq w - c^w$ , then there is an equilibrium where each firm charges  $p^w = w - \frac{t}{2n}$  and serves  $\frac{1}{n}$ th of the market. Standard arguments show that there exists no other symmetric equilibrium outcome. **Step 4.** We derive the symmetric equilibrium outcome when  $w - c^w < \frac{t}{n}$ . Consider any firm  $i$  and suppose that it is a monopolist. If it charges the price  $p_i^w$ , the distance  $d$  to the consumer who is indifferent between trading or not is defined by  $w - p_i^w - td = 0$ . Demand for firm  $i$ 's  $w$ -product is then  $D_i = \frac{2(w - p_i^w)}{t}$ , from which we obtain the optimal price  $p^w = \frac{w + c^w}{2}$ . At this price, the marginal consumers for firm  $i$  are located less than  $\frac{1}{2n}$  away from it if and only

if  $w - c^w < \frac{t}{n}$ . By construction, if this inequality holds, it is optimal for all firms to charge  $p^w = \frac{w+c^w}{2}$ . Standard arguments show that this is also the unique equilibrium outcome. This completes the proof.  $\square$

*Proof of Proposition 2, statements (i), (ii), (iv) and (v).* We prove the four statements in steps.

**Step 1.** We prove statement (i). Suppose that  $\frac{t}{n} < p_{add}^v - c^v$  and consider an assessment where firms shroud add-on prices, all rational consumers purchase the  $w$ -product at price  $p^w = c^w + \frac{t}{n}$ , and all boundedly rational consumers purchase the  $v$ -product at price  $p^v = p_{add}^v$  (which means that the base price of the  $v$ -product is zero). As discussed in the main text, this implies that boundedly rational consumers strictly prefer the  $v$ -product from firm  $i$  to the  $w$ -product from firm  $i$ . The assumption on the maximal add-on price ensures that no consumer earns a negative payoff. Note that, in this assessment, firms earn strictly more from selling a  $v$ -product than from selling a  $w$ -product. Thus, no firm can gain by educating consumers. Standard arguments show that firms also cannot profit from charging different prices. Hence, the considered assessment is an equilibrium. **Step 2.** We prove statement (ii). Suppose that  $p_{add}^v - c^v \leq \frac{t}{n} \leq \frac{2}{3}(v - c^v)$  and  $p_{add}^v > (w - c^w) - (v - c^v)$ . Consider an assessment where firms shroud add-on prices, all rational consumers purchase the  $w$ -product at price  $p^w = c^w + \frac{t}{n}$ , and all boundedly rational consumers purchase the  $v$ -product at price  $p^v = c^v + \frac{t}{n}$  (they charge the maximal add-on price). The assumption on  $p_{add}^v$  ensures that boundedly rational consumers strictly prefer the  $v$ -product from firm  $i$  to the  $w$ -product from firm  $i$ . The assumption on  $\frac{t}{n}$  ensures that no consumer earns a negative payoff. Firms earn the same profit from trading the  $w$ -product and the  $v$ -product. Prices are determined by competition between firms as in the symmetric benchmark equilibrium from Proposition 1. Thus, no firm can gain by educating consumers or by charging different prices. Hence, the considered assessment is an equilibrium. **Step 3.** We prove statement (iv). Suppose that  $\frac{2}{3}(w - c^w) < \frac{t}{n} \leq w - c^w$ . Assume by contradiction that there is a symmetric equilibrium in which firms trade the  $v$ -product. As in the proof of statement (iii) of Proposition 2 below, we can show that, in this equilibrium, firms must sell the  $w$ -product to rational consumers and the  $v$ -product to boundedly rational consumers. Note from Proposition 1 that the symmetric equilibrium profits from trading the  $v$ -product are strictly below the symmetric equilibrium profits from trading the  $w$ -product. Hence, the equilibrium price  $p^w$  for the  $w$ -product must weakly exceed the symmetric benchmark equilibrium price  $w - \frac{t}{2n}$ . If it were lower, each firm would have an incentive to increase this price (note that it could do so without disturbing trade with boundedly rational consumers). Consequently, each firm could profitably deviate by educating consumers and selling the  $w$ -product to all consumers at the symmetric benchmark equilibrium price, a contradiction. The last part of statement (iv) follows from standard arguments. Finally, the proof of statement (v) is straightforward and therefore omitted.  $\square$

*Proof of Proposition 2, statement (iii).* Assume by contradiction that an equilibrium exists in which the  $v$ -product is traded. We find a contradiction in several steps. **Step 1.** We show that in an equilibrium in which the  $v$ -product is traded, each firm also sells the  $w$ -product to a positive share of rational consumers. Observe that the  $v$ -product can only be traded in equilibrium when all firms shroud their add-on prices (otherwise, all consumers would be rational and only the  $w$ -product would survive competitive pressure and profit maximization). Assume by contradiction that there is an equilibrium in which firm  $i$  only sells the  $v$ -product at total price  $p_i^v$  to consumers. In equilibrium, we must have that firm  $i$  serves a positive fraction of rational as well as boundedly rational consumers, and earns a positive profit so that  $p_i^v > c^v$ . Suppose firm  $i$  now offers the  $w$ -product at total price

$$p_i^w = p_i^v - (v - w) - \epsilon. \quad (14)$$

If  $\epsilon$  is small enough, the following holds: At least those rational consumers who trade with firm  $i$  now purchase the  $w$ -product instead of the  $v$ -product. Since  $w - c^w > v - c^v$ , this increases firm  $i$ 's total profit, a contradiction. **Step 2.** We make two important observations. First, note that at any given position on the circle, it cannot happen in equilibrium that a boundedly rational consumer purchases a  $w$ -product, while a rational consumer at the same position purchases a  $v$ -product. The reason is that the boundedly rational consumer would then strictly underestimate the total price of the  $v$ -product and switch to this product. Second, observe that, by the tie-breaking rule, in any equilibrium in which the  $v$ -product is traded, all boundedly rational consumers who trade with firm  $i$  purchase the  $w$ -product if  $w - p_{i,base}^w \geq v - p_{i,base}^v$ , and purchase the  $v$ -product if  $w - p_{i,base}^w < v - p_{i,base}^v$ . **Step 3.** We show that, in an equilibrium in which the  $v$ -product is traded, there is no firm  $i$  that charges a price  $p_i^w$  below the rational benchmark level  $c^w + \frac{t}{n}$ . Assume by contradiction that there exists an equilibrium in which firm  $i$  charges  $p_i^w < c^w + \frac{t}{n}$ ; w.l.o.g. we assume that firm  $i$  offers the  $w$ -product at the lowest price,  $p_i^w \leq p_j^w$  for any other firm  $j$ . Let firms  $j, k$  be firm  $i$ 's neighbors. By Step 1 and Step 2, firm  $i$ 's profit then equals

$$\pi_i = \lambda(p_i^u - c^u)(D_{ij}^{br} + D_{ik}^{br}) + (1 - \lambda)(p_i^w - c^w)(D_{ij}^r + D_{ik}^r), \quad (15)$$

where  $u \in \{w, v\}$  denotes the product that boundedly rational consumers trade with firm  $i$  (according to Step 2), and  $D_{ij}^r$  ( $D_{ij}^{br}$ ) denotes the mass of rational (boundedly rational) consumers who trade with firm  $i$  and who are located on the interval between firm  $i$  and firm  $j$ ;  $D_{ik}^r$ ,  $D_{ik}^{br}$  denote the same for consumers located on the interval between firm  $i$  and firm  $k$ . In Step 4 to Step 7, we show that either firm  $i$  can increase  $\pi_i$  by charging a higher price for the  $w$ -product or one of firm  $i$ 's neighbors can increase its profit by charging a different price for the  $w$ -product.

**Step 4.** Suppose there is a boundedly rational consumer who in equilibrium (i) purchases the  $w$ -product from firm  $i$ , (ii) is indifferent between the  $w$ -product from firm  $i$  and the  $v$ -product from firm  $j$  or firm  $k$ , and (iii) strictly prefers trading to not trading. By Step 2, statement (i) implies that firm  $i$  then only sells the  $w$ -product to consumers. Note that the consumer overestimates the utility from purchasing the  $v$ -product. Hence, by continuity and statements (ii) and (iii), firm  $i$  could increase its profit by advertising add-on prices. The described situation therefore cannot occur in equilibrium. **Step 5.** Suppose the marginal consumers who purchase the  $w$ -product from firm  $i$  (i) are indifferent between the  $w$ -product from firm  $i$  and the  $w$ -product from firm  $j$  or firm  $k$ , and (ii) strictly prefer trading to not trading. We show that in this case firm  $i$  can increase its profit by raising  $p_i^w$  by some small  $\epsilon > 0$ . The marginal consumer on the segment between firm  $i$  and firm  $j$  is defined by the equality

$$w - p_i^w - \epsilon - td = w - p_j^w - t\left(\frac{1}{n} - d\right), \quad (16)$$

and hence by

$$d = \frac{p_j^w - p_i^w - \epsilon}{2t} + \frac{1}{2n}. \quad (17)$$

The profit that firm  $i$  is making from selling the  $w$ -product to consumers located between firm  $i$  and firm  $j$  (normalized by the share of rational/boundedly rational consumers) equals

$$\tilde{\pi}_i = (p_i^w + \epsilon - c^w) \left( \frac{p_j^w - p_i^w - \epsilon}{2t} + \frac{1}{2n} \right). \quad (18)$$

Observe that  $\tilde{\pi}_i$  strictly increases in  $\epsilon$  if

$$-p_j^w + 2(p_i^w + \epsilon) < c^w + \frac{t}{n}. \quad (19)$$

By construction, we have  $p_i^w \leq p_j^w$ , which implies that this inequality is satisfied if  $\epsilon$  is sufficiently small. Finally, note that increasing  $p_i^w$  by a small amount would not affect firm  $i$ 's sales of its  $v$ -product, which completes the proof of the statement. **Step 6.** Suppose that, in equilibrium, there exists a firm  $j$  that charges a price  $p_j^w > c^w + \frac{t}{n}$  so that the marginal consumers who purchase the  $w$ -product from firm  $j$  are indifferent between trading or not trading. We show that it cannot be the case that the boundedly rational consumers who trade with firm  $j$  (i) purchase the  $v$ -product, (ii) are arbitrary close to indifference between purchasing the  $w$ -product and the  $v$ -product from firm  $j$ , and (iii) firm  $j$  earns weakly more out of boundedly rational consumers than out of rational consumers. Assume by contradiction that this is the case. Since boundedly rational consumers who purchase the  $v$ -product overestimate the price of the  $w$ -product, statement (ii) implies that the marginal boundedly rational consumer who

purchases the  $v$ -product is closer to firm  $j$  than the marginal rational consumer who purchases the  $w$ -product. The fact that  $\frac{t}{n} < \frac{2}{3}(w - c^w)$  and  $w - c^w > v - c^v$  then imply that statement (iii) cannot be true. **Step 7.** Suppose the marginal consumer on the segment between firm  $i$  and firm  $j$  who purchases the  $w$ -product from firm  $i$  is indifferent between this product and not trading at all. This implies that the marginal consumer on the same segment who purchases the  $w$ -product from firm  $j$  is also indifferent between this product and not trading at all. Moreover, since firm  $i$  charges the lowest price for the  $w$ -product, this also implies that the marginal consumer on the segment between firm  $j$  and its other neighbor, firm  $j'$ , who purchases the  $w$ -product from firm  $j$  is also indifferent between this product and not trading at all. Note that firm  $j$ 's marginal consumers for the  $w$ -product are located less than  $\frac{1}{2n}$  away from firm  $j$ . We show that firm  $j$  then can increase its profit by lowering its price  $p_j^w$  by some small  $\epsilon$ . Since  $\frac{t}{n} < \frac{2}{3}(w - c^w)$ , this is true if after the price adjustment the marginal consumers are still indifferent between trading or not trading the  $w$ -product with firm  $j$ . Assume therefore w.l.o.g. that after the price adjustment the marginal consumer on the segment between firm  $j$  and firm  $j'$  is defined by

$$w - p_j^w + \epsilon - td = w - p_{j'}^w - t\left(\frac{1}{n} - d\right). \quad (20)$$

The profit that firm  $j$  is making from selling the  $w$ -product to consumers located between firm  $j$  and firm  $j'$  (normalized by the share of rational/boundedly rational consumers) equals

$$\tilde{\pi}_j = (p_j^w - \epsilon - c^w) \left( \frac{p_{j'}^w - p_j^w + \epsilon}{2t} + \frac{1}{2n} \right). \quad (21)$$

We can show that  $\tilde{\pi}_j$  strictly increases in  $\epsilon$  if

$$2(p_j^w - \epsilon) - p_{j'}^w > c^w + \frac{t}{n}. \quad (22)$$

The location of marginal consumers implies that  $p_{j'}^w < p_j^w$ . Hence, the inequality in (22) is satisfied if  $\epsilon$  is sufficiently small. The result then follows from Step 6 when we apply the same steps to the segment between firm  $i$  and firm  $j$  and take into account that firm  $j$  can also choose to educate boundedly rational consumers. Taken together, Step 5 and Step 7 imply that, in an equilibrium in which the  $v$ -product is traded, there is no firm  $i$  that charges a price  $p_i^w < c^w + \frac{t}{n}$ .

**Step 8.** We show that an equilibrium in which the  $v$ -product is traded cannot exist. Consider first any firm  $i$  that only sells the  $w$ -product to consumers (if such a firm exists). Note that the share of boundedly rational consumers it serves must be at least as large as the share of rational consumers,  $D_{ij}^{br} + D_{ik}^{br} \geq D_{ij}^r + D_{ik}^r$ , otherwise firm  $i$  could gain by educating consumers. Moreover, since all of firm  $i$ 's rivals charge at least  $c^w + \frac{t}{n}$  for the  $w$ -product, the optimal price for firm  $i$  is such that the fraction rational consumers it serves is at least  $\frac{1}{n}$ . Next, consider any

firm  $j$  that sells the  $v$ -product to boundedly rational consumers. Since all firms charge at least  $c^w + \frac{t}{n}$  for the  $w$ -product, this firm can earn at least the symmetric benchmark equilibrium profit out of rational consumers. Since  $\frac{t}{n} > \frac{2}{3}(v - c^v)$ , it earns weakly more out of boundedly rational consumers only if the fraction of boundedly rational consumers it serves exceeds  $\frac{1}{n}$ . By the observations above, this cannot be the case for all firms that sell the  $v$ -product. Hence, at least one of them could profitably educate consumers and charge different prices. This completes the proof.  $\square$

*Proof of Proposition 3.* The statement that firm 1 does not invest into any type of innovation if all consumers are rational directly follows from the firm profits in the benchmark equilibrium as described in Proposition 1. Statement (i) follows from the fact that in the symmetric shrouding equilibrium and in any symmetric equilibrium with advertised add-on prices the markup on the  $w$ -product is  $\frac{t}{n}$ , so that the firms' profits from the  $w$ -product are independent of  $w - c^w$ . We prove statement (ii). Suppose that firm 1 conducts the  $v$ -innovation project. If  $\tilde{v} - \tilde{c}^v > w - c^w$ , then, given the prices in the symmetric shrouding equilibrium, any firm  $i$  may have an incentive to advertise add-on prices and to sell only the (now superior)  $v$ -product. We derive the optimal total price for this deviation, assuming that all other firms charge the symmetric shrouding equilibrium prices. When all consumers are rational, they strictly prefer the  $w$ -product from a firm  $j \neq i$  to the  $v$ -product from firm  $j$ . We derive the maximal price  $p_i^v$  firm  $i$  can charge for the  $v$ -product so that it serves (almost) all consumers. If  $n$  is odd, this price equals  $p_i^v = \tilde{v} - (w - c^w) - \frac{t}{n} \frac{n+1}{2}$ , and the corresponding profit (if positive) equals

$$\pi_i = (\tilde{v} - \tilde{c}^v) - (w - c^w) - \frac{t}{n} \frac{n+1}{2}. \quad (23)$$

If  $n$  is even, this price equals  $p_i^v = \tilde{v} - (w - c^w) - \frac{t}{n} \frac{n+2}{2}$ , and the corresponding profit (if positive) equals

$$\pi_i = (\tilde{v} - \tilde{c}^v) - (w - c^w) - \frac{t}{n} \frac{n+2}{2}. \quad (24)$$

The optimal total price for the  $v$ -product may be larger than these prices. Define by  $x \geq 1$  the number of segments between firm  $i$  and the firm furthest away from firm  $i$  so that firm  $i$  still serves consumers in segment  $x$ . If  $n$  is odd, the maximal value of  $x$  equals  $\frac{n-1}{2}$ , and if  $n$  is even, the maximal value of  $x$  equals  $\frac{n}{2}$ . If firm  $i$  charges a price  $p_i^v$  so that it does not serve all consumers, then the marginal consumers is characterized by

$$\tilde{v} - p_i^v - td = w - \left(c^w + \frac{t}{n}\right) - t\left(\frac{x}{n} - d\right), \quad (25)$$

and firm  $i$ 's profit equals

$$\pi_i = (p_i^v - \tilde{c}^v) \left( \frac{(\tilde{v} - p_i^v) - (w - c^w)}{t} + \frac{1}{n}(1 + x) \right). \quad (26)$$

Thus, if the price  $p_i^v$  is optimal, it satisfies

$$p_i^v = \frac{\tilde{v} - w + \tilde{c}^v + c^w}{2} + \frac{t}{2n}(1 + x), \quad (27)$$

and the corresponding profit equals

$$\pi_i = \frac{1}{t} \left[ \frac{(\tilde{v} - \tilde{c}^v) - (w - c^w)}{2} + \frac{t}{2n}(1 + x) \right]^2, \quad (28)$$

where  $x$  is the largest value so that, at the beginning of segment  $x$ , consumers prefer the  $v$ -product of firm  $i$  to the  $w$ -product of their closest firm, i.e.,

$$x = \left\lfloor \left[ \frac{(\tilde{v} - \tilde{c}^v) - (w - c^w)}{t} \right] \frac{n}{t} + 1 \right\rfloor. \quad (29)$$

The equations (23), (24), and (28) define the maximal profit  $\pi^{mm}(\tilde{v}, \tilde{c}^v)$  from the proposed deviation. We can verify that  $\pi^{mm}(\tilde{v}, \tilde{c}^v) = 0$  if  $\tilde{v} - \tilde{c}^v = w - c^w$ ,  $\pi^{mm}(\tilde{v}, \tilde{c}^v)$  strictly increases in  $\tilde{v} - \tilde{c}^v$ , and that, as  $\tilde{v} - \tilde{c}^v$  increases, there is at most one intersection with  $\tilde{\pi}^{sh} = \frac{1}{n} \left[ \lambda \left( \tilde{v} - \frac{t}{2n} - \tilde{c}^v \right) + (1 - \lambda) \frac{t}{n} \right]$ , the profit from compliance in the symmetric shrouding equilibrium. As soon as the proposed deviation becomes profitable, the symmetric shrouding equilibrium no longer exists, and products are traded with markup  $\frac{t}{n}$  in any symmetric continuation equilibrium. From this statement (ii) follows.  $\square$

## A.2 The Model without Education

We consider the model from Section 2, but assume that firms cannot advertise add-on prices to educate consumers. Firms only set prices and the behavior of boundedly rational consumers must be a personal equilibrium according to Definition 1. We analyze the set of equilibria in this new game and obtain the following result.

**Proposition 5** (Equilibrium in the Model without Education). *Suppose firms cannot advertise add-on prices. Assume that there is a share of boundedly rational consumers,  $\sqrt{\frac{1}{2}}(v - c^v) \leq \frac{2}{3}(w - c^w)$ , and that the maximal add-on price is small enough such that  $v - p_{add}^v \geq \frac{1}{3}(v - c^v)$ .*

- (i) *If  $\frac{t}{n} < v - p_{add}^v$  and  $p_{add}^v > (w - c^w) - (v - c^v)$ , then (a) there exists no equilibrium in which only the  $w$ -product is traded, and (b) in the unique symmetric equilibrium,*

rational consumers purchase the  $w$ -product at price  $p^w = c^w + \frac{t}{n}$  and boundedly rational consumers purchase the  $v$ -product at price  $p^v = p_{add}^v$ .

(ii) If  $v - p_{add}^v \leq \frac{t}{n} < \frac{2}{3}(v - c^v)$  and  $p_{add}^v > (w - c^w) - (v - c^v)$ , then (a) there exists no equilibrium in which only the  $w$ -product is traded, and (b) in the unique symmetric equilibrium, rational consumers purchase the  $w$ -product at price  $p^w = c^w + \frac{t}{n}$  and boundedly rational consumers purchase the  $v$ -product at price  $p^v = c^v + \frac{t}{n}$ .

(iii) If  $\frac{2}{3}(v - c^v) \leq \frac{t}{n} < \sqrt{\frac{1}{2}}(v - c^v)$  and  $p_{add}^v > (w - c^w) - \frac{5}{6}(v - c^v)$ , then (a) there exists no equilibrium in which only the  $w$ -product is traded, and (b) in the unique symmetric equilibrium, rational consumers purchase the  $w$ -product at price  $p^w = c^w + \frac{t}{n}$  and boundedly rational consumers purchase the  $v$ -product at price  $p^v = v - \frac{t}{2n}$ .

(iv) If  $\sqrt{\frac{1}{2}}(v - c^v) \leq \frac{t}{n}$ , there exists a symmetric equilibrium in which all consumers purchase the  $w$ -product at the prices indicated in Proposition 1. This is also the equilibrium that maximizes industry profits.

The proof of Proposition 5 is presented below. The two qualifications at the beginning of the proposition are again not essential, they only save us from more case distinctions. Proposition 5 contains a number of important observations. First, firms may sell only the superior product to consumers even though add-on prices remain shrouded. Specifically, this can happen when firms have sufficient market power, see statement (iv). Starting from a situation where all firms sell the  $w$ -product at symmetric equilibrium prices, a single firm then cannot profit from selling the inferior product. Intuitively, its appeal to boundedly rational consumers does not outweigh the reduced gains from trade. Hence, market power holds back firms from introducing inferior products.

When the market is sufficiently competitive and the inferior product appears sufficiently attractive to boundedly rational consumers relative to the superior product (i.e.,  $p_{add}^v$  is large enough), such an equilibrium does not exist, see statements (i) to (iii). In a situation where only the  $w$ -product is sold, each firm would have an incentive to introduce the  $v$ -product to exploit its advantage with boundedly rational consumers.

Second, in a symmetric equilibrium, firms benefit from the existence of the  $v$ -product only if the market is sufficiently competitive, i.e., when  $\frac{t}{n} < v - p_{add}^v$ , see statement (i). Only then are symmetric equilibrium profits strictly larger than in the benchmark equilibrium. For intermediate degrees of competition, firms either earn the same profit as in the symmetric benchmark equilibrium, see statement (ii), or strictly less, see statement (iii). Hence, firms have incentives to introduce the  $v$ -product only if competition is fierce enough.

Overall, the differences between the model with costless education and the model without education are modest. With costless education, there always exists an equilibrium in which only the  $w$ -product is traded, while without education this is only the case when there is sufficiently little competition. It seems plausible that firms coordinate on an equilibrium where they benefit from shrouding equilibrium prices if such benefits exist in equilibrium. Further, with costless education, the  $v$ -product is no longer traded when there is sufficiently little competition between firms. In contrast, without education it still can be optimal for firms to sell the inferior product to boundedly rational consumers, even when they enjoy a local monopoly. The reason is that boundedly rational consumers can have pessimistic beliefs in a personal equilibrium when firms only offer the  $w$ -product. These consumers then overestimate the probability paying an add-on charge after purchasing the  $w$ -product and therefore may prefer not trading at all to purchasing the  $w$ -product. Not learning the true probability of add-on charges after trading the  $w$ -product then leaves these beliefs unchallenged.

*Proof of Proposition 5.* We prove statements (ii) to (iv) in steps. The proof of statement (i) uses very similar arguments as the proof of statement (ii) in Steps 1 and 2 and is therefore omitted. **Step 1.** We show that if  $v - p_{add}^v \leq \frac{t}{n} < \frac{2}{3}(v - c^v)$  and  $p_{add}^v > (w - c^w) - (v - c^v)$ , then there exists no equilibrium in which only the  $w$ -product is traded. By Proposition 1, each firm would charge  $p^w = c^w + \frac{t}{n}$  and earn total profits of  $\pi = \frac{t}{n^2}$  in such an equilibrium. We show that a firm then can deviate profitably by selling the  $v$ -product to some boundedly rational consumers. Suppose firm  $i$  sells the  $v$ -product at base price  $p_{i,base}^v$ . All boundedly rational consumers at a distance  $d \leq \frac{1}{2n}$  to firm  $i$  weakly prefer the  $v$ -product from firm  $i$  to the  $w$ -product from firm  $i$  if

$$v - p_{i,base}^v - p_{add}^v \geq w - \left(c^w + \frac{t}{n}\right) - p_{add}^v, \quad (30)$$

which can be rewritten as

$$p_{i,base}^v \leq v + \frac{t}{n} - (w - c^w). \quad (31)$$

These consumers also weakly prefer to purchase the  $v$ -product from firm  $i$  to not purchasing anything if

$$v - p_{i,base}^v - p_{add}^v - \frac{t}{2n} \geq 0, \quad (32)$$

which can be re-arranged as

$$p_{i,base}^v \leq v - \frac{t}{2n} - p_{add}^v. \quad (33)$$

Suppose firm  $i$  sells the  $v$ -product at the base price defined by the right-hand side of inequality (31). The profit from boundedly rational consumers is then larger than under the original

situation where only the  $w$ -product is sold if

$$\left( (v - c^v) - (w - c^w) + \frac{t}{n} + p_{add}^v \right) \frac{1}{n} > \frac{t}{n^2}, \quad (34)$$

which is equivalent to the assumption that  $p_{add}^v > (w - c^w) - (v - c^v)$ . Next, assume that the base price is defined by the right-hand side of inequality (33). Firm  $i$ 's profit from boundedly rational consumers is then larger than under the original situation where only the  $w$ -product is sold if

$$\left( v - c^v - \frac{t}{2n} \right) \frac{1}{n} > \frac{t}{n^2}, \quad (35)$$

which is equivalent to the assumption that  $\frac{t}{n} < \frac{2}{3}(v - c^v)$ . The two observations taken together imply that there exists a profitable deviation for firm  $i$ , which completes the proof of the statement. **Step 2.** We show that if  $v - p_{add}^v \leq \frac{t}{n} < \frac{2}{3}(v - c^v)$  and  $p_{add}^v > (w - c^w) - (v - c^v)$ , then there exists a symmetric equilibrium in which rational consumers purchase the  $w$ -product at price  $p^w = c^w + \frac{t}{n}$  and boundedly rational consumers purchase the  $v$ -product at price  $p^v = c^v + \frac{t}{n}$ . This statement directly follow from statement (ii) of Proposition 2 as in the present situation firms can no longer educate consumers. **Step 3.** We show that if  $\frac{2}{3}(v - c^v) \leq \frac{t}{n} < \sqrt{\frac{1}{2}}(v - c^v)$  and  $p_{add}^v > (w - c^w) - \frac{5}{6}(v - c^v)$ , then there exists no equilibrium in which only the  $w$ -product is traded. By Proposition 1 and the assumption that  $\sqrt{\frac{1}{2}}(v - c^v) \leq \frac{2}{3}(w - c^w)$ , each firm would charge  $p^w = c^w + \frac{t}{n}$  and earn total profits of  $\pi = \frac{t}{n^2}$  in such an equilibrium. We show that firm  $i$  can deviate profitably by selling the  $v$ -product to boundedly rational consumers. Suppose firm  $i$  charges the base price  $p_{i,base}^v$  for the  $v$ -product, sells it to all boundedly rational consumers up to distance  $d$ , and that the marginal boundedly rational consumer is indifferent between purchasing the  $v$ -product from firm  $i$  and not trading at all. We then have

$$v - p_{i,base}^v - p_{add}^v - td = 0, \quad (36)$$

so that  $p_{i,base}^v = v - p_{add}^v - td$ . This deviation is profitable if

$$2(v - c^v - td)d > \frac{t}{n^2}. \quad (37)$$

From this inequality we get the critical distance

$$d^* = \frac{(v - c^v) + \frac{1}{2} \sqrt{(v - c^v)^2 - 2 \left( \frac{t}{n} \right)^2}}{2t}. \quad (38)$$

There is a profitable deviation where firm  $i$  sells the  $v$ -product to boundedly rational consumers if, at  $p_{i,base}^v = v - p_{add}^v - td^*$ , the boundedly rational consumers at distance  $d^*$  (i) strictly prefer

the  $v$ -product from firm  $i$  to the  $w$ -product from the neighboring firm and (ii) strictly prefer the  $v$ -product from firm  $i$  to the  $w$ -product from firm  $i$ . Condition (i) holds if

$$v - p_{i,base}^v - p_{add}^v - td^* > w - \left(c^w + \frac{t}{n}\right) - p_{add}^v - t\left(\frac{1}{n} - d^*\right). \quad (39)$$

This inequality is equivalent to

$$p_{add}^v > (w - c^w) + \frac{1}{2}(v - c^v) - \frac{2t}{n}. \quad (40)$$

Since  $\frac{t}{n} \geq \frac{2}{3}(v - c^v)$ , this inequality is implied by the assumption on  $p_{add}^v$ . Condition (ii) holds if

$$v - p_{i,base}^v > w - \left(c^w + \frac{t}{n}\right). \quad (41)$$

Again, this inequality follows from  $\frac{t}{n} \geq \frac{2}{3}(v - c^v)$  and the assumption on  $p_{add}^v$ . This completes the proof of the statement. **Step 4.** We show that if  $\frac{2}{3}(v - c^v) \leq \frac{t}{n} < \sqrt{\frac{1}{2}}(v - c^v)$  and  $p_{add}^v > (w - c^w) - \frac{5}{6}(v - c^v)$ , then there exists a symmetric equilibrium in which rational consumers purchase the  $w$ -product at price  $p^w = c^w + \frac{t}{n}$  and boundedly rational consumers purchase the  $v$ -product at price  $p^v = v - \frac{t}{2n}$ . The assumption on  $p_{add}^v$  ensures that, at these prices, boundedly rational consumers indeed purchase the  $v$ -product. The only potentially profitable deviation of any firm  $i$  is to increase the base price of the  $v$ -product such that boundedly rational consumers prefer the  $w$ -product of firm  $i$  to the  $v$ -product of firm  $i$ . We show that no such deviation is profitable by deriving an upper bound on the profit that firm  $i$  can make from selling the  $w$ -product to boundedly rational consumers. To get this upper bound, we assume that firm  $i$  can price discriminate between rational and boundedly rational consumers. We distinguish between two cases, Case (i) and Case (ii). Case (i): Assume that under the optimal base price  $p_{i,base}^w$  charged to boundedly rational consumers, the marginal boundedly rational consumer is indifferent between purchasing the  $w$ -product from firm  $i$  and not trading at all. The boundedly rational consumers' beliefs imply that the marginal consumers are then characterized by the equality

$$w - p_{i,base}^w - p_{add}^v - td = 0. \quad (42)$$

The profit from selling the  $w$ -product to boundedly rational consumers is then

$$\pi_i = 2(p_{i,base}^w - c^w) \left( \frac{w - p_{i,base}^w - p_{add}^v}{t} \right). \quad (43)$$

The price that maximizes this profit is  $p_{i,base}^w = \frac{w+c^w}{2} - \frac{p_{add}^v}{2}$ , and the corresponding profit equals

$$\pi_i = \frac{2}{t} \left( \frac{w - c^w - p_{add}^v}{2} \right)^2. \quad (44)$$

We compare this profit to the profit firm  $i$  would make from boundedly rational consumers on the equilibrium path. The deviation is not profitable if

$$\left( \frac{w - c^w - p_{add}^v}{2} \right)^2 \leq \frac{1}{2} \frac{t}{n} (v - c^v) - \frac{1}{4} \left( \frac{t}{n} \right)^2. \quad (45)$$

From the assumption on the marginal boundedly rational consumer we get

$$\frac{w - p_{i,base}^w - p_{add}^v}{t} \leq \frac{1}{2n}, \quad (46)$$

which implies that  $p_{add}^v \geq w - c^w - \frac{t}{n}$ . We use this inequality to re-write (45) and get that it is satisfied if  $\frac{t}{n} \leq v - c^v$ . By assumption, we have  $\frac{t}{n} \leq \sqrt{\frac{1}{2}}(v - c^v)$ , which proves the statement for Case (i). Case (ii): Assume that under the optimal base price  $p_{i,base}^w$  charged to boundedly rational consumers, the marginal boundedly rational consumer is indifferent between purchasing the  $w$ -product from firm  $i$  and purchasing the  $v$ -product from a neighboring firm. The marginal consumer is then characterized by the equality

$$w - p_{i,base}^w - td = v - \left( c^v + \frac{t}{n} - p_{add}^v \right) - t \left( \frac{1}{n} - d \right). \quad (47)$$

So the marginal consumer is located at

$$d = \frac{(w - p_{i,base}^w) - (v - c^v) - p_{add}^v}{2t} + \frac{1}{n}. \quad (48)$$

The profit from selling the  $w$ -product to boundedly rational consumers is then

$$\pi_i = 2(p_{i,base}^w - c^w) \left( \frac{(w - p_{i,base}^w) - (v - c^v) - p_{add}^v}{2t} + \frac{1}{n} \right). \quad (49)$$

The price that maximizes this profit is  $p_{i,base}^w = \frac{t}{n} + \frac{w+c^w}{2} - \frac{v-c^v}{2} - p_{add}^v$ , and the corresponding profit equals

$$\pi_i = \frac{1}{t} \left( \frac{t}{n} + \frac{(w - c^w) - (v - c^v) - p_{add}^v}{2} \right)^2. \quad (50)$$

We compare this profit to the profit firm  $i$  would make from boundedly rational consumers on

the equilibrium path. The deviation is not profitable if

$$\left(\frac{t}{n} + \frac{(w - c^w) - (v - c^v) - p_{add}^v}{2}\right)^2 \leq \frac{t}{n}(v - c^v) - \frac{1}{2}\left(\frac{t}{n}\right)^2. \quad (51)$$

The assumption on  $p_{add}^v$  and  $\frac{t}{n} < \sqrt{\frac{1}{2}}(v - c^v)$  ensure that this inequality is satisfied, which proves the statement for Case (ii). **Step 5.** We show that if  $\sqrt{\frac{1}{2}}(v - c^v) \leq \frac{t}{n}$ , then there exists a symmetric equilibrium in which all consumers purchase the  $w$ -product at the prices indicated in Proposition 1. Assume first that  $\frac{t}{n} \leq \frac{2}{3}(w - c^w)$ . Consider an assessment where all firms charge  $p^w = c^w + \frac{t}{n}$  for the  $w$ -product, a base price for the  $v$ -product above  $v$  (so that no consumer purchases it), and all consumers purchase the  $w$ -product. We show that no firm can profitably deviate from this assessment by changing the price of the  $v$ -product. Suppose firm  $i$  reduces the price of the  $v$ -product so that boundedly rational consumers purchase it. In the best case, the marginal boundedly rational consumers are indifferent between purchasing the  $v$ -product from firm  $i$  and not trading at all. These consumers would be characterized by

$$v - p_{i,base}^v - p_{add}^v - td = 0. \quad (52)$$

The optimal base price would then be  $p_{i,base}^v = \frac{v+c^v}{2} - p_{add}^v$ , and the corresponding profit from boundedly rational consumers (normalized by their share)

$$\pi_i = \frac{(v - c^v)^2}{2t}. \quad (53)$$

We compare this profit to the profit firm  $i$  makes from boundedly rational consumers under the original assessment. By Proposition 1, this value is  $\frac{t}{n^2}$ . Since  $\sqrt{\frac{1}{2}}(v - c^v) \leq \frac{t}{n}$ , there is no profitable deviation, which proves the claim for  $\frac{t}{n} \leq \frac{2}{3}(w - c^w)$ . Similarly, we can show that there exists a symmetric equilibrium in which all consumers purchase the  $w$ -product at the prices indicated in Proposition 1 when  $w - c^w > \frac{t}{n} \geq \frac{2}{3}(w - c^w)$  and when  $\frac{t}{n} > w - c^w$ . In the former case, this follows from the assumption that  $\sqrt{\frac{1}{2}}(v - c^v) \leq \frac{2}{3}(w - c^w)$ . Finally, the last statement follows from the fact that firms sell the  $w$ -product at the equilibrium price that maximizes industry profits.  $\square$

### A.3 Omitted Proofs from Section 4

*Proof of Lemma 1.* We prove statement (i). Assume that the marginal rational and boundedly rational consumers are indifferent between the two neighboring firms and weakly prefer trading to not trading. If all other firms charge the base price  $p_{-i,base}$ , then demand for

firm  $i$ 's  $v$ -product from both rational and boundedly rational consumers at base price  $p_{i,base}$  is  $D_i = \frac{p_{-i,base} - p_{i,base}}{t} + \frac{1}{n}$ , and firm  $i$ 's profit equals

$$\pi_i = \lambda(p_{i,base} + p_{add} - c) \left( \frac{p_{-i,base} - p_{i,base}}{t} + \frac{1}{n} \right) + (1 - \lambda)(p_{i,base} - c) \left( \frac{p_{-i,base} - p_{i,base}}{t} + \frac{1}{n} \right). \quad (54)$$

From this we get that, in a symmetric equilibrium, each firm charges the base price  $p_{base} = c - \lambda p_{add} + \frac{t}{n}$ . We check until what value of transport costs all consumers indeed purchase the product at this price. Since  $p_{add} > e$ , this is the case if

$$v - p_{base} - p_{add} - \frac{t}{2n} \geq 0, \quad (55)$$

which is equivalent to

$$\frac{2}{3}[(v - c) - (1 - \lambda)p_{add}] \geq \frac{t}{n}. \quad (56)$$

This proves the result. The uniqueness of the symmetric equilibrium outcome follows from standard arguments which show that at a higher (lower) base price  $p_{base}$  firms have an incentive to cut (increase) the base price. This also holds for the other statements and will not be repeated. We prove statement (ii). Suppose that  $\frac{t}{n}$  is large enough so that it violates the threshold in (56). We show that if  $\frac{t}{n}$  is not too large, then, in a symmetric equilibrium, firms charge a base price so that they serve all consumers, and the marginal boundedly rational consumers are indifferent between trading and not trading. By definition, this base price must be equal to  $p_{base} = v - p_{add} - \frac{t}{2n}$ . If firm  $i$  deviates and increases the price by some small  $\epsilon > 0$ , its profit equals

$$\pi_i = 2\lambda(p_{base} + p_{add} + \epsilon - c) \frac{v - p_{base} - p_{add} - \epsilon}{t} + (1 - \lambda)(p_{base} + \epsilon - c) \left( \frac{1}{n} - \frac{\epsilon}{t} \right). \quad (57)$$

Differentiating this expression with respect to  $\epsilon$  gets us that firms have no incentive to charge a higher price if

$$\frac{2}{3 + \lambda} [(1 + \lambda)(v - c) - (1 - \lambda)p_{add}] \geq \frac{t}{n}. \quad (58)$$

Standard arguments show that firms also cannot profitably charge a lower price, which completes the proof of statement (ii). We prove statement (iii). Suppose that  $\frac{t}{n}$  is large enough so that it violates the threshold in (58). Assume that the marginal rational consumers are indifferent between the two neighboring firms and strictly prefer trading to not trading, while the marginal boundedly rational consumers are indifferent between trading with the closest firm and not trading at all. We derive the symmetric equilibrium price for this setting. Firm  $i$ 's

profit from charging  $p_{i,base}$  when all other firms charge  $p_{-i,base}$  equals

$$\pi_i = 2\lambda(p_{i,base} + p_{add} - c) \frac{v - p_{i,base} - p_{add}}{t} + (1 - \lambda)(p_{i,base} - c) \left( \frac{p_{-i,base} - p_{i,base}}{t} + \frac{1}{n} \right). \quad (59)$$

From the corresponding first-order condition we get that, in the unique symmetric equilibrium, firms charge

$$p_{base} = \frac{2\lambda(v + c - 2p_{add}) + (1 - \lambda)(c + \frac{t}{n})}{1 + 3\lambda}. \quad (60)$$

We can check that under this price the marginal boundedly rational consumers are indeed indifferent between trading with the closest firm and not trading at all, given that (58) is violated. The assumption on  $p_{add}$  ensures that some boundedly rational consumers still purchase the  $v$ -product under the base price in (60). Finally, we check until what value of transport costs all rational consumers purchase the product at this price. This is the case if

$$v - p_{base} - e - \frac{t}{2n} \geq 0, \quad (61)$$

which is equivalent to

$$\frac{2}{3 + \lambda} [(1 + \lambda)(v - c) + 4\lambda p_{add} - (1 + 3\lambda)e] \geq \frac{t}{n}. \quad (62)$$

This completes the proof of statement (iii). We prove statement (iv). Suppose that  $\frac{t}{n}$  is large enough so that it violates the threshold in (62). We show that if  $\frac{t}{n}$  is not too large, then in a symmetric equilibrium firms charge a base price so that they serve all rational consumers, and the marginal rational and boundedly rational consumers are indifferent between trading or not trading. By definition, this base price must be equal to  $p_{base} = v - e - \frac{t}{2n}$ . The assumption on  $p_{add}$  ensures that some boundedly rational consumers still purchase the  $v$ -product under this price. If firm  $i$  deviates and increases the price by some small  $\epsilon > 0$ , its profit equals

$$\pi_i = 2\lambda(p_{base} + p_{add} + \epsilon - c) \frac{v - p_{base} - p_{add} - \epsilon}{t} + 2(1 - \lambda)(p_{base} + \epsilon - c) \frac{v - p_{base} - e - \epsilon}{t}. \quad (63)$$

Differentiating this expression with respect to  $\epsilon$  gets us that firms have no incentive to charge a higher price if

$$(v - c) + 2\lambda p_{add} - (1 + \lambda)e \geq \frac{t}{n}. \quad (64)$$

Standard arguments show that firms also cannot profitably charge a higher price, which completes the proof of statement (vi). We prove statement (v). Suppose  $\frac{t}{n}$  is large enough so that it violates the threshold in (64). Assume that the marginal rational and boundedly rational consumers are indifferent between trading with the closest firm and not trading at all. We derive

the equilibrium price for this setting. Firm  $i$ 's profit from charging  $p_{i,base}$  equals

$$\pi_i = 2\lambda(p_{i,base} + p_{add} - c)\frac{v - p_{base} - p_{add}}{t} + 2(1 - \lambda)(p_{i,base} - c)\frac{v - p_{i,base} - e}{t}. \quad (65)$$

From the corresponding first-order condition we get that, in the unique symmetric equilibrium, firms charge

$$p_{base} = \frac{v + c}{2} - \frac{2\lambda p_{add} + (1 - \lambda)e}{2}. \quad (66)$$

We can check that under this price the marginal rational and boundedly rational consumers are indeed indifferent between trading with the closest firm and not trading at all, given that (64) is violated. The assumption on  $p_{add}$  ensures that some boundedly rational consumers still purchase the  $v$ -product under the price in (60), which completes the proof of statement (v).  $\square$

*Proof of Proposition 4.* We prove statement (i). Assume that it is indeed optimal for firms to shroud add-on prices. By Lemma 1 and the assumption  $\frac{2}{3}[(v - c) - p_{add}] \geq \frac{t}{n}$ , we must have that, in a symmetric shrouding equilibrium, firms charge the base price  $p_{base} = c - \lambda p_{add} + \frac{t}{n}$ , the maximal add-on price  $p_{add}$ , and each firm earns a profit of

$$\pi^{sh} = \frac{t}{n^2}. \quad (67)$$

Suppose firm  $i$  unshrouds its add-on price and charges w.l.o.g. an add-on price of zero. We derive the optimal base-price. If firm  $i$  charges the base price  $p_{i,base}$  and this base price is not too large, the marginal consumers for firm  $i$  are defined by

$$v - p_{i,base} - td = v - p_{base} - t\left(\frac{1}{n} - d\right). \quad (68)$$

The share of consumers it serves then equals  $D_i = \frac{c - \lambda p_{add} + \frac{t}{n} + e - p_{i,base}}{t} + \frac{1}{n}$ . Its profit from charging  $p_{i,base}$  is  $\pi_i = (p_{i,base} - c)D_i$ , and the optimal base price equals

$$\tilde{p}_{i,base} = c + \frac{t}{n} - \frac{\lambda p_{add} - e}{2}. \quad (69)$$

Hence, the highest possible profit from deviation is

$$\pi^{un} = \left(\frac{t}{n} - \frac{\lambda p_{add} - e}{2}\right)\left(\frac{1}{n} - \frac{\lambda p_{add} - e}{2t}\right). \quad (70)$$

From equations (67) and (70) we get that no firm can deviate profitably from the suggested symmetric shrouding equilibrium if  $\lambda p_{add} \geq e$ , which completes the proof of the statement. We prove statement (ii). We prove it separately for three cases: Case (i)  $\frac{t}{n} < \frac{2}{3}(v - c)$ , Case (ii)

$\frac{2}{3}(v - c) \leq \frac{t}{n} < v - c$ , and Case (iii)  $v - c \leq \frac{t}{n} < (v - c) + p_{add} - e$ . Consider Case (i). Suppose that  $\lambda \approx 1$  and that it is indeed optimal for firms to shroud add-on prices. Lemma 1 then shows that, in a symmetric shrouding equilibrium, firms charge the base price  $p_{base} \approx c - p_{add} + \frac{t}{n}$ , the maximal add-on price  $p_{add}$ , and each firm serves the share  $\frac{1}{n}$  of rational and boundedly rational consumers. As in the proof of statement (i) we compute the optimal base price of firm  $i$  after unshrouding the add-on price. At  $\lambda \approx 1$  it equals

$$\tilde{p}_{i,base} \approx c - \frac{p_{add} - e}{2} + \frac{t}{n} \quad (71)$$

given that the add-on price is set to zero. This value is strictly smaller than  $p_{base} + p_{add}$ . The marginal consumers are then at distance

$$d \approx \frac{1}{2n} - \frac{\tilde{p}_{i,base} - (c - p_{add} + \frac{t}{n})}{2t} < \frac{1}{2n} \quad (72)$$

to their closest firm. Hence, by continuity, if  $\lambda$  is sufficiently large, a firm cannot profitably unshroud its add-on price. Next, consider Case (iii). Suppose that  $\lambda \approx 1$  and that it is indeed optimal for firms to shroud add-on prices. Lemma 1 then shows that, in a symmetric shrouding equilibrium, firms charge the base price  $p_{base} \approx \frac{v+c}{2} - p_{add}$ , the maximal add-on price  $p_{add}$ , and hence earn approximately monopoly profits. Marginal boundedly rational consumers are located at distance

$$d \approx \frac{v - c}{2t}. \quad (73)$$

If firm  $i$  unshrouds its add-on price and charges the same total price as before, then for  $\lambda \approx 1$  the marginal boundedly rational consumer (and those boundedly rational consumers close to her) trade with firm  $i$ 's rivals if

$$v - p_{base} - e - t \left( \frac{1}{n} - \frac{v - c}{2t} \right) > 0, \quad (74)$$

which for  $\lambda = 1$  is equivalent to

$$\frac{t}{n} < (v - c) + p_{add} - e. \quad (75)$$

Hence, if  $\lambda$  is close enough to 1, then a firm cannot profitably unshroud its add-on price as its profit would be bounded away from monopoly profits. Finally, the proof for Case (ii) is very similar to that for Case (iii) and therefore omitted. We prove statement (iii). Assume by contradiction that a shrouding equilibrium exists. In this equilibrium, firms charge the maximal add-on price as well as the base price  $p_{base} = \frac{v+c}{2} - \frac{2\lambda p_{add} + (1-\lambda)e}{2}$  if it is optimal for them to serve both rational and boundedly rational consumers, and  $p_{base} = \frac{v+c}{2} - \frac{e}{2}$  if it is optimal for them to

serve only rational consumers. Moreover, rational consumers exert substitution effort in this equilibrium. Assume for a moment that firm  $i$  is the only firm in the market and all consumers are boundedly rational. It would then earn maximal profits by unshrouding its add-on price and charging  $p_{i,base} = \frac{v+c}{2}$  and  $p_{i,add} = 0$ . The sum of these prices would also be the unique optimal total price. Next, assume that firm  $i$  is the only firm in the market and all consumers are rational. Again, firm  $i$  would then earn the maximal profit by unshrouding its add-on price and charging the same prices (and their sum would again be the unique optimal total price). Taking these observations together implies that firm  $i$  can profitably deviate by unshrouding its add-on price and charging  $p_{i,base} = \frac{v+c}{2}$  and  $p_{i,add} = 0$ , a contradiction.  $\square$