Competitive Markets and Boundedly Rational Expectations*

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Abstract

We analyze the trade of differentiated products when firms set both base and potentially hidden add-on prices. Boundedly rational consumers mistakenly believe that their action (product choice or substitution effort) has no effect on the probability of paying add-on prices, but all consumers correctly anticipate their equilibrium expenses. The model is thus applicable to markets for high-frequency products or markets where firms wish to avoid "surprise charges" due to reputational concerns. Shrouding equilibria with inefficient trade exist in this setting, but only if the market is sufficiently competitive. The presence of boundedly rational consumers can generate innovation incentives that improve welfare relative to the rational consumer benchmark. For credit/debit card markets the model explains why informational interventions have only minor effects on behavior, while add-on price regulation increases consumer surplus.

Keywords: Competition, Add-On Pricing, Bounded Rationality
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1 Introduction

Many products are offered with complex pricing schedules that comprise both base prices and add-on charges for additional services. Classic examples are credit or debit cards, mobile-phone services, and actively managed mutual funds. Markets for such products have a reputation for hiding add-on prices so that unsuspecting consumers can be overcharged. A significant literature suggests that competition between firms is not enough to make prices transparent, e.g., Gabaix and Laibson (2006), Armstrong and Vickers (2012), and Heidhues et al. (2017). The argument is that firms can exploit "myopic" (or "naive") consumers who do not take add-on prices into account when making purchase decisions. Educating these consumers about add-on prices would make them unprofitable for firms. Hence, firms have no incentives to make pricing schedules transparent. Consequently, there is a strong case for public policy to educate consumers or to enforce transparent price schedules through disclosure requirements.

The myopic consumer argument is, however, not entirely convincing in some markets. Credit and debit cards are "high-frequency products" (Gathergood et al. 2020) that provide quick feedback to consumers. Credit card fees are made salient on credit card statements. Overcharge fees for debit card transactions are also communicated quickly to consumers. Since consumers have to manage their finances continuously, it does not seem plausible that a large fraction of them remains unaware about interest charges and penalty fees for a long time. Indeed, the empirical research on consumers' use of credit and debit cards provides little indication that their behavior changes substantially when interest charges and penalites are highlighted through interventions.[1]

Feedback on outcomes is less frequent for actively managed funds. Nevertheless, the myopic consumer argument also has issues for these products. Actively managed funds are typically traded through brokers or financial advisors. Consumers need to trust their broker or advisor to make investments into risky assets (Gennaioli et al. 2015). They follow the asset managers whom they trust (Kostovetsky 2016), and a reduction in trust reduces investments in equity (Guiso et al. 2008, Gurun et al. 2018, Choi and Robertson 2020). The myopic consumer argument ignores trust and reputation concerns. In the myopic consumer world, the ongoing trade of mutual funds with high add-on charges would continuously surprise consumers and produce disappointed (less wealthy than expected) customer generations. This is at odds with an industry that is very concerned with reputation and which exists since many decades.

In this paper, we consider a market for products with complex pricing schedules in which some consumer are boundedly rational, but where all consumers correctly anticipate the distribution over their expenses in equilibrium. We therefore avoid the conflict between consumer

[1] We describe this research in detail in Section 5.

expectations and actual experiences that is difficult to justify for high-frequency products or trade that requires consumer trust. The model generates new results on the role of competition and regulation in markets for products with complex pricing schedules. It provides a natural prediction for maximal add-on prices and it offers a new perspective on the welfare implications of innovation in such markets. Finally, the model provides explanations for recent empirical findings in credit/debit card markets as well as in the market for mutual funds.

To discipline our approach, we directly build on the multi-product version of Heidhues et al. (2017, henceforth HKM17), and, at a later stage, also on Gabaix and Laibson (2006, henceforth GL06). As in these papers, firms set base and add-on prices, and can choose whether to educate consumers by advertising add-on prices. There are rational and boundedly rational consumers. The latter ones become rational when at least one firm advertises add-on prices. To allow for competition effects, we assume that products are horizontally differentiated. Importantly, the only significant change we make relative to HKM17 and GL06 is the consumers' belief formation process, which we model by using Spiegler's (2016) Bayesian network approach. All consumers observe or correctly anticipate the firms' pricing schedule. A consumer's action – product choice in the HKM17 setting and substitution effort in the GL06 framework – affects her probability of paying an add-on charge. Rational consumers understand this link. Boundedly rational consumers mistakenly believe that their action has no influence on the probability of paying the add-on charge. When they select an action in equilibrium that frequently triggers the billing of additional charges, they extrapolate that this is also the case for all alternative actions. Therefore, they may undervalue products where add-ons are charged infrequently or underestimate the benefits of substitution effort. The change in the belief formation process generates the following results.

Shrouding equilibria and competition. We first study under what circumstances a shrouding equilibrium with inefficient trade exists in our framework. Inefficient trade means that boundedly rational consumers purchase an inferior product (in the HKM17 setting) or that rational consumers exert substitution effort to avoid add-on charges (in the GL06 setting). We find that shrouding equilibria with inefficient trade exist in our framework, but only if the market is sufficiently competitive. This result is intuitive. As the market becomes less competitive, firms reap a larger share of the gains from trade. This creates incentives to maximize these gains, and hence to avoid the inefficiencies that occur when add-on prices are shrouded. In our HKM17 setting, this implies that firms only sell the superior product to consumers if competition is sufficiently relaxed.[2] In our GL06 setting, firms advertise their add-on prices to prevent rational consumers from exerting substitution effort if firms enjoy sufficient market power.

[2]We show in an extension that this also holds if firms cannot educate consumers by advertising add-on prices. The assumption of free (and effective) consumer education is therefore not essential.

Crucially, these findings rely on the fact that all consumers correctly anticipate their equilibrium expenses. If boundedly rational consumers would ignore add-on prices and add-on prices were large enough, then a monopolist would happily exploit them to earn profits beyond the monopoly level. Overall, we conclude that that the main results in HKM17 and GL06 do not rely on differences between consumers' expectations and actual experiences in equilibrium. Therefore, they can be consistent with high-frequency products and reputation building. However, if consumers correctly anticipate their equilibrium expenses, the shrouding equilibria in these settings only obtain if the competition between firms is sufficiently intense.

Endogenous maximal add-on prices. The model generates a natural prediction for the maximal add-on price. In a myopic consumer model, this variable is exogenously given and may reflect the level of consumer protection provided through regulation. Since myopic consumers do not take add-on prices into account, there is no bound on these prices. In our framework, if the maximal add-on price is too high, a shrouding equilibrium would not exist since boundedly rational consumers would refuse to trade with firms to avoid exploitation. Our version of the HKM17 framework allows to characterize an "optimal add-on price", i.e., the maximal add-on price that maximizes industry profits. It is well-defined if the market is sufficiently competitive, and then equals the price firms would choose in an optimal cartel if all consumers were boundedly rational. We then can show that, under the optimal add-on price, the gap in surplus for rational and boundedly rational consumers widens as the market becomes more competitive. Furthermore, it can "poison" boundedly rational consumers' beliefs in the sense that these consumers strictly prefer no trade to purchasing the superior product.

Incentives for welfare-improving innovation. We obtain new results on firms' innovation incentives. Heidhues et al. (2016, henceforth HKM16) show that, in a shrouding equilibrium with non-appropriable innovation (firms can freely copy others' findings), firms always have incentives to invest into exploitative innovation that increases the maximal add-on price, but no incentives to invest into product innovation which increases the payoff from the product that myopic consumers purchase.

This changes significantly with boundedly rational equilibrium beliefs. First, incentives for exploitative innovation are bounded since firms would not push the maximal add-on price above its optimal level. Second, if firms can keep the maximal add-on price at the optimal level through continuous exploitative innovation, they directly benefit from product innovations that increase the value of the inferior product to consumers. Third, in a shrouding equilibrium, firms' profits directly depend on the production costs of the inferior product. Hence, they are willing to invest into process innovation that reduces these production costs. Fourth, both product and process innovation are not limited to the elimination of welfare losses from inefficient trade. We show that the shrouding equilibrium survives if the inferior product becomes the superior product as long as the gains from trade of the two products are not too different. Therefore, the presence of boundedly rational consumers can generate market power that eventually improves market welfare relative to the rational consumer benchmark.

Implications for regulation. Our model yields several implications for the regulation of markets with complex pricing schedules. First, interventions that inform consumers about add-on charges may not change behavior since consumers already are aware of them. An informational intervention is only effective if it updates consumers' understanding of how alternative behaviors would affect the probability of add-on charges. Most likely, this is difficult to achieve through simple nudging or disclosure policies. Next, add-on price regulation can be effective. This measure can redistribute surplus back to boundedly rational consumers. We show that lowering maximal add-on prices also reduces the scope for shrouding equilibria. However, our results on innovation imply that there can be a trade-off between welfare and consumer surplus. If maximal add-on prices are limited through regulation, then, in a shrouding equilibrium, firms have no incentives to invest into non-appropriable product innovation.

Explanations for empirical findings on markets with complex pricing schedules. The model provides an explanation for several empirical patterns that are difficult to rationalize with both rational decision making and myopic beliefs. For credit/debit card markets, the model explains why informational interventions have only minor effects on consumer behavior, while add-on price regulation improves consumer surplus. A number of studies analyzed the effect of informational interventions on consumer behavior. These results were highly anticipated in the context of low consumer financial literacy and abusive practices in retail finance. However, the informational interventions showed only small or insignificant effects (e.g., Seira et al. 2017). In contrast, the regulation of credit card fees had substantial effects on consumer surplus (Agarwal et al. 2015). Our model offers a uniform explanation for both observations.

For the market for mutual funds, there is an ongoing debate on why a share of consumers purchases actively managed mutual funds although it is well known that, after fees, most of these products perform worse than passive index funds. Moreover, many consumers seek recommendations from financial advisors who steer them towards these products. However, the existence of add-on fees is typically made salient to consumers, either because financial advisors mention them anyway during the consultation (Mullainathan et al. 2012), or because regulation such as the European "Markets in Financial Instruments Directive" forces firms to make all costs transparent. Our model shows that this does not threaten the existence of shrouding equilibria with inefficient trade as long as boundedly rational consumers erroneously extrapolate the cost structure of the recommended product to other products.

Related Literature. The paper contributes to the literature on add-on pricing in competitive markets. Verboven (1999) and Ellison (2005) consider settings with only rational consumers. High add-on prices then can emerge if firms cannot advertise these prices. Since in many markets (such as credit/debit card markets) this assumption is not satisfied, GL06 consider add-on pricing with myopic consumers. They show that welfare is wasted in a shrouded attributes equilibrium as rational consumers exert substitution effort, while myopic consumers end up paying high add-on prices. A number of papers study similar settings with myopic or naive consumers, see Grubb (2009, 2014), Miao (2010), Armstrong and Vickers (2012), Inderst and Ottaviani (2012), Dahremöller (2013), Shulman and Geng (2013), Warren and Wood (2014), Kosfeld and Schüwer (2017), Ko and Williams (2017), and Johnen (2020). HKM17 consider a version of the GL06 setting in which welfare is wasted since myopic consumers trade inferior products. These welfare losses are arguably more severe than the losses from costly substitution effort. In this paper, we replace myopic beliefs by boundedly rational equilibrium beliefs to avoiding the conflict between consumer expectations and experiences. In contrast to the previous literature, we show that the presence of boundedly rational consumers creates innovation incentives that may eventually increase welfare.

The paper is also related to the literature on price obfuscation, see Carlin (2009), Piccione and Spiegler (2012), and Chioveanu and Zhou (2013). The major difference between models of price obfuscation and hidden add-on pricing is that in the former firms can take an action to change the share of boundedly rational consumers. Specifically, firms can "confuse" some consumers by adopting non-transparent pricing schedules. These consumers then make random purchase decisions or buy some default product. Carlin (2009) and Chioveanu and Zhou (2013) also find a link between price obfucation and competition: The equilibrium degree of confusion increases in the number of firms. Importantly, to obtain this result, these models make the assumption that firms cannot charge prices above the consumers' valuation for the product. This ensures that random purchase is weakly better than no purchase at all. Without such an assumption, maximum confusion and unlimited prices would be optimal at any degree of competition. Our approach allows to avoid such an assumption while still obtaining a link between competition and obfuscation.

To model boundedly rational beliefs, we use the Bayesian network framework from Spiegler (2016). This framework offers a non-parametric, portable model of belief formation when the decision-maker applies misspecified causal reasoning to make sense out of the data that she gets in equilibrium. Recent papers apply the Bayesian network framework to study monetary policy (Spiegler 2020), political competition (Eliaz and Spiegler 2020), Bayesian persuasion (Eliaz et al. 2021), decision making (Schenone 2020, Ellis and Thysen 2021), and incentive contracts (Schumacher and Thysen 2021).

The rest of the paper is organized as follows. In Section 2, we introduce our baseline model which builds on the HKM17 framework with multiple products. In Section 3, we analyze under what circumstances there exists an equilibrium in which firms shroud add-on prices and sell the inferior product to consumers. We derive optimal maximal add-on prices and study firms' innovation incentives. In Section 4, we consider a version of our model that builds on the GL06 framework with substitution effort. In Section 5, we discuss how the model explains recent findings in markets for credit and debit card markets as well as in mutual fund markets. Section 6 concludes. All proofs and mathematical details are relegated to the appendix.

2 Model

We analyze a market with horizontal product differentiation, superior and inferior products, as well as rational and boundedly rational consumers. To this end, we combine Salop's (1979) model of product differentiation with the multi-product market model from HKM17, and Spiegler's (2016) Bayesian network framework.

Basic Framework. There is a unit mass of consumers. They are located uniformly on a circle with a perimeter equal to 1. Each consumer wants to buy at most one unit of a good. There are n firms $i = 1, ..., n$ located around the circle with equal distance between them. Each firm offers two products, a w-product and a v-product. A w-product generates utility w for a consumer and has unit production costs c^w, a v-product generates utility v and has unit costs c^v. While a v-product generates more utility than a w-product, $v > w$, it is still inferior since it is more costly to produce, that is, we have $w - c^w > v - c^v$. If a consumer trades a good of utility u with a firm that is at distance d to her and pays the price p, her utility equals $u - p - td$.

Price-States and Prices. Denote by $a \in A = \{0, 1, 2\}$ whether a consumer chooses a w-product ($a = 2$), a v-product ($a = 1$), or no product at all ($a = 0$). The price she pays for the product depends on the "price-state" x_3, which can be high ($x_3 = 2$), low ($x_3 = 1$), or zero ($x_3 = 0$). The price-state captures how the product is used and constitutes the legal basis for charging the consumer. It can be interpreted as the list of items on the final receipt: The high price-state represents a receipt with charges for many items, the low price-state is a receipt with only a charge for the base good, and the zero price-state is the case when the consumer does not receive any invoice. The w-product is designed so that its consumption mostly leads to the low price-state, while consuming the v-product mostly leads to the high price-state. In the following paragraph, we explain the causal relationship between choices and price states.

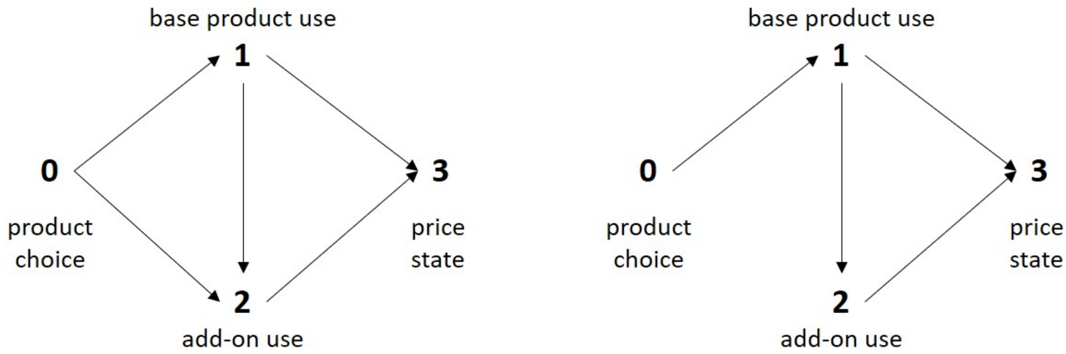

Figure 1: Objective model $\mathcal{R}^*$ (left) and subjective model $\mathcal{R}$ (right).

The causal structure of the transaction $\mathcal{R}^*$ is shown on the left graph of Figure 1. The probability of price-state x_3 depends on the use of a base good $x_1 \in \{0, 1\}$ and the use of an add-on $x_2 \in \{0, 1\}$; we denote it by $q(x_3 \mid x_1, x_2)$. This conditional distribution has full support for any x_1, x_2, but the high price-state is likely if both base good and add-on are used, $q(x_3 = 2 \mid x_1 = 1, x_2 = 1) \approx 1$, the low price-state is likely if only the base good is used, $q(x_3 = 1 \mid x_1 = 1, x_2 = 0) \approx 1$, and the zero price-state is likely if the base good is not used, $q(x_3 = 0 \mid x_1 = 0, x_2) \approx 1$. The consumer's product choice influences whether the base good and the add-on are used. The use of the add-on also depends on base good usage. Let $q(x_1 \mid a)$ and $q(x_1 \mid a, x_1)$ denote the corresponding conditional probabilities. Again, they always have full support, but base good usage is likely if some product is chosen, and unlikely otherwise. In particular, we have $q(x_1 = 1 \mid a = 2) = q(x_1 = 1 \mid a = 1) \approx 1$. Add-on usage is likely if the v-product or no product is chosen ($a \in 0, 1$) and the base good is used ($x_1 = 1$), otherwise it is unlikely. Specifically, we have $q(x_2 = 1 \mid a = 0, x_1 = 1) = q(x_2 = 1 \mid a = 1, x_1 = 1) \approx 1$. This assumption is not essential for our main results, but it simplifies the analysis and the interpretation of the model substantially.[3] As indicated above, the high price-state is likely if the consumer chooses the v-product, the low price-state is likely if she chooses the w-product, and the zero price-state is likely if she does not trade.

Firm i chooses for each product $u \in \{v, w\}$ a first price $p^u_{i,1}$ and a second price $p^u_{i,2}$. Both must be non-negative and the maximal second price is $\bar{p} > 0$. In the low price-state, the price of the u-product equals $p^u_{i,1}$. In the high price-state, it is $p^u_{i,1} + p^u_{i,2}$. Let $a(u)$ be the action that chooses the u-product, $a(w) = 2$ and $a(v) = 1$. For convenience, we will use the following terminology. Firm i's base price for the u-product is

$$p^u_{i,base} = q(x_3 = 1 \mid a(u))\, p^u_{i,1} + q(x_3 = 2 \mid a(u))\, p^u_{i,1}. \tag{1}$$

[3] A possible intuition is as follows: Consumers sometimes get invoices for products they did not purchase (which of course they do not have to pay). These invoices are typically inflated by additional charges.

and its add-on price for this product is

$$p^u_{i,add} = q(x_3 = 2 \mid a(u))\ p^u_{i,2}. \tag{2}$$

Firm i's total price for the u-product is $p^u_i = p^u_{i,base} + p^u_{i,add}$. Denote the maximal add-on price for the u-product by $p^u_{add} = q(x_3 = 2 \mid a(u))\bar{p}$, and the difference between the maximal add-on prices by $\tilde{p} = p^v_{add} - p^w_{add}$. We assume $p^w_{add} < c^w$. Firm i's profit from each product u is the mass of consumers who purchase this product from firm i times the markup $p^u_i - c^u$.

Shrouded Prices and Boundedly Rational Consumers. Firms can shroud add-on prices or advertise them.[4] If firm i shrouds its add-on prices, consumers only observe $p^w_{i,1}, p^v_{i,1}$ and assume $p^w_{i,2} = p^v_{i,2} = \bar{p}$. If it advertises its add-on prices, consumers observe $p^w_{i,1}, p^v_{i,1}, p^w_{i,2}, p^v_{i,2}$.

Consumers differ in their understanding of what influences the probability distribution over price-states. Rational consumers correctly anticipate the association between product choice and price-states $q(x_3 \mid a)$. Boundedly rational consumers neglect the influence of product choice on the probability of add-on usage. Their subjective causal model is given by the graph $\mathcal{R}$ on the right of Figure 1. They fit this model to the equilibrium joint probability distribution $q(a, x_1, x_2, x_3)$ generated by their equilibrium strategy $q \in \Delta(A)$ to derive subjective beliefs $q_{\mathcal{R}}(x_3 \mid a; q)$ about the distribution over price-states when action a is chosen. In Appendix A.1, we explicitly derive $q_{\mathcal{R}}(x_3 \mid a; q)$ for any q. Boundedly rational beliefs have three important properties. First, at any given equilibrium strategy q, boundedly rational consumers correctly anticipate the equilibrium distribution over price-states, that is

$$\sum_{a \in A} q(a) q_{\mathcal{R}}(x_3 \mid a; q) = \sum_{a \in A} q(a) q(x_3 \mid a) \tag{3}$$

for each $x_3 \in \{0, 1, 2\}$. Their equilibrium payoff will therefore never fall below their reservation utility of zero. Second, boundedly rational consumers believe that the distribution over price-states is independent of product choice, that is

$$q_{\mathcal{R}}(x_3 \mid a = 1; q) = q_{\mathcal{R}}(x_3 \mid a = 2; q) \tag{4}$$

for any equilibrium strategy q. The intuition for this is as follows: The association between product choice and base good usage is the same for both products. According to the subjective model $\mathcal{R}$, the remaining variables (add-on usage and price-state) are independent from product choice for a given realization of base-good usage. Hence, to a boundedly rational consumer, product choice appears as having no effect on the distribution over price-states. Finally, the

[4] To maintain consistency with the literature, we use "add-on price" instead of "second price."

assumption that add-on usage is equally likely when a consumer chooses the v-product and no product at all implies that boundedly rational consumers have *pessimistic* beliefs: If their equilibrium strategy is "no trade with certainty", they think that the high price-state is likely if they purchase any product,

$$q_{\mathcal{R}}(x_3 = 2 \mid a; q(0) = 1) = q(x_3 = 2 \mid a = 1) \approx 1 \tag{5}$$

for each $a \in \{1, 2\}$. A boundedly rational consumer's expected payoff $U_{\mathcal{R}}(a \mid q)$ from action a when her equilibrium strategy is q equals $U_{\mathcal{R}}(0 \mid q) = 0$ and

$$U_{\mathcal{R}}(a \mid q) = \max_i \left[u(a) - q_{\mathcal{R}}(x_3 = 1 \mid a; q) p_{i,1}^{u(a)} - q_{\mathcal{R}}(x_3 = 2 \mid a; q)(p_{i,1}^{u(a)} + p_{i,2}^{u(a)}) - td_i \right] \tag{6}$$

for $a \in \{1, 2\}$, where d_i is the consumer's distance to firm i, $u(a)$ is the inverse of $a(u)$, and $p_{i,2}^{u(a)} = \bar{p}$ if firm i shrouds add-on prices. Since subjective beliefs depend on the equilibrium strategy q, we adopt the personal equilibrium concept from Spiegler (2016):

Definition 1. *The strategy q is a personal equilibrium for a boundedly rational consumer at given and expected prices if $a \in \arg\max_{a'} U_{\mathcal{R}}(a' \mid q)$ for all actions $a \in A$ in the support of q, and $q_{\mathcal{R}}(x_3 \mid a'; q) = \lim_{k \to \infty} q_{\mathcal{R}}(x_3 \mid a'; q^k)$ for all actions $a' \in A$ and a sequence $q^k \to q$ of fully mixed strategy profiles.*

We can now fully describe the model and solution concept. First, firms simultaneously choose first and second prices as well as whether to shroud or advertise add-on prices. If at least one firm advertises its add-on prices, all consumers are rational.[5] Otherwise, the share λ of consumers is boundedly rational. After observing first and second prices, consumers choose the product-firm combination that yields them the highest expected payoff according to their beliefs. The choice of a boundedly rational consumer must be a personal equilibrium. We assume that, in case of a tie between firms, each optimal firm is chosen with equal probability, and in case of a tie between w-product and v-product, consumers choose the w-product.[6] In equilibrium, each firm maximizes its profit given the rivals' and consumers' strategies, the rational consumers' strategies are optimal for them at given (expected) prices, and the boundedly rational consumers' strategies are personal equilibria at given (expected) prices.

This equilibrium definition rules out that firms surprise consumers with unexpected overcharges. The implicit assumption here is that such overcharges are unprofitable as they hurt the firms' reputation and reduce future sales. The model thus captures in particular product markets with frequent consumer feedback or markets where trade requires consumer trust.

[5]Our main results do not depend on this extreme assumption, see the discussion in Subsection 3.6.

[6]This rules out equilibria with mixed consumer strategies.

While using the Bayesian network framework uses more notation than a short-cut model of biased beliefs, it has some crucial advantages. First, it ensures that beliefs are always derived from the underlying process of the transaction, which makes it portable to other contexts. Second, the Bayesian network model makes a consumer's causal reasoning explicit so that we can examine how different misperceptions affect her beliefs. In Appendix A.1, we further elaborate on this topic. Third, the Bayesian network framework provides a unifying structure for models of boundedly rational beliefs, see Section 5 in Spiegler (2016). One can therefore make precise how the belief bias in the present framework differs from that in other models with biased beliefs. Finally, as we will see next, our model with boundedly rational equilibrium beliefs is as convenient to handle as a classic IO model.

3 Equilibrium Trade of Superior and Inferior Products

In this section, we derive the main results of our model. In Subsection 3.1, we examine the benchmark cases when there are only rational consumers or when only the inferior product is available. In Subsection 3.2, we state our main result. It describes under what circumstances a shrouding equilibrium with inefficient trade exists. In Subsection 3.3, we use this result to characterize the add-on price that maximizes industry profits, that is, the "optimal add-on price." In Subsection 3.4, we consider exploitative innovation in our framework and show that it may establish the optimal add-on price in equilibrium. In Subsection 3.5, we further use this extension to examine firms' incentives to invest into product and process innovation. In Subsection 3.6, we briefly discuss how the main result would change if it is impossible for firms to educate consumers.

3.1 The Benchmark Equilibrium

We first examine the equilibrium outcome when all consumers are rational. In this case, there is no scope for trade of the inferior product. Since the gains from trading the v-product are smaller than the gains from trading the w-product, it is not profitable for firms to sell the inferior product, regardless of the intensity of competition. Therefore, only the w-product is traded in equilibrium. The following result describes the market outcome – prices and profits – in the symmetric benchmark equilibrium.

Proposition 1 (Benchmark Equilibrium)**.** *Suppose all consumers are rational. Then only the w-product is traded in equilibrium. The unique symmetric equilibrium outcome is as follows:*

(i) If $\frac{t}{n} < \frac{2}{3}(w - c^w)$, firms share the market equally, each firm charges $p^w = c^w + \frac{t}{n}$ and earns total profits of $\pi = \frac{t}{n^2}$; this is also the unique equilibrium outcome.

(ii) If $\frac{2}{3}(w-c^w) \leq \frac{t}{n} \leq w-c^w$, firms share the market equally, each firm charges $p^w = w - \frac{t}{2n}$ and earns total profits of $\pi = \frac{w-c^w}{n} - \frac{t}{2n^2}$.

(iii) If $w - c^w < \frac{t}{n}$, each firm enjoys a local monopoly and serves less than $\frac{1}{n}$th of the market; it charges $p^w = \frac{w+c^w}{2}$ and earns total profits of $\pi = \frac{(w-c^w)^2}{2t}$; this is also the unique equilibrium outcome.

The proof of Proposition 1 is in Appendix A.2. We will frequently refer to this result throughout the paper. Case (i) is the standard case that is discussed in the literature on product differentiation. The degree of product differentiation is small enough such that firms are competing against each other and set prices so that the marginal consumers (those who are located in the middle between two neighboring firms) earn a positive surplus. In this domain, a firm's individual profit equals $\frac{t}{n^2}$ and thus strictly increases in transport costs. In Case (ii), the degree of product differentiation is large enough so that, in the symmetric equilibrium, firms are no longer competing against each other. Instead, they charge prices that make the consumers located in the middle between two neighboring firms indifferent between trading (with either firm) or not trading at all. These consumers become more difficult to serve as transport costs increase. Hence, in this domain, a firm's individual profit strictly decreases in transport costs. Finally, in Case (iii), the degree of product differentiation is sufficiently large so that it no longer pays off for firms to serve all consumers. Each firm enjoys a local monopoly and the marginal consumers are indifferent between trading with the closest firm and not trading at all. Again, individual firm profits are decreasing in transport costs in this domain. Figure 2 displays the firms' individual profits in the symmetric equilibrium for the three cases.

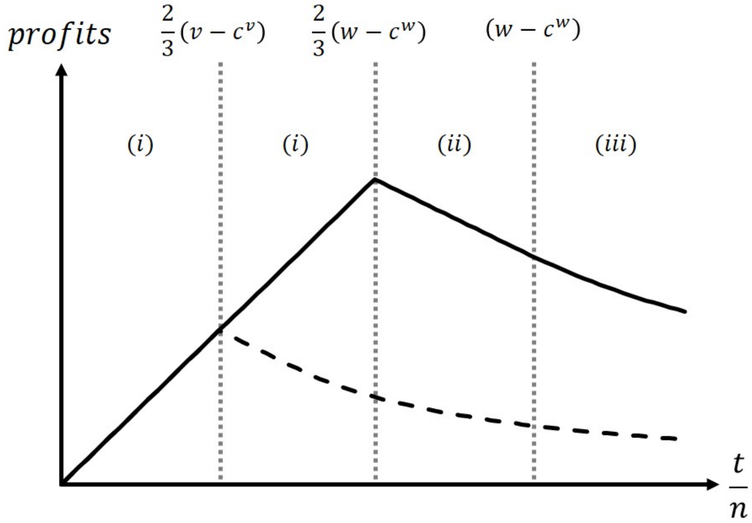

Figure 2: Individual firm profits from trading the w-product in the symmetric benchmark equilibrium (solid black line), and individual firm profits from trading the v-product in the symmetric equilibrium (dashed black line) when only the v-product is available. The numbers (i) to (iii) indicate the cases from Proposition 1.

Before we analyze the general case, we consider a further useful benchmark. Suppose that there are now both rational and boundedly rational consumers, and that only the v-product is available. If consumers can only choose between trading and not trading the v-product, the beliefs of both consumer types are identical and rational, $q_{\mathcal{R}}(x_3 \mid a; q) = q(x_3 \mid a)$ for any $a \in \{0, 1\}$ and q. Proposition 1 then also indicates the symmetric equilibrium outcomes for this benchmark: We only have to replace $w - c^w$ by $v - c^v$ in the statement. Note that at a low degree of product differentiation, i.e., when $\frac{t}{n} \leq \frac{2}{3}(v - c^v)$, the symmetric equilibrium profits in the two benchmark cases are identical. In this domain, competition is intense enough so that markups are determined only by transport costs, while the gains from trade, $w - c^w$ and $v - c^v$, play no role for firm profits. In contrast, when the degree of product differentiation is large enough, $\frac{t}{n} > \frac{2}{3}(v - c^v)$, then the symmetric equilibrium profits from trading only the w-product strictly exceed those from trading only the v-product. Figure 2 illustrates this difference through the dashed line which indicates the symmetric equilibrium profits from trading the v-product.

3.2 The Equilibrium with Boundedly Rational Consumers

We examine the market equilibrium when some consumers are boundedly rational. Note that the outcome from the benchmark equilibrium is always an equilibrium outcome. When all firms advertise their add-on prices and act as in the benchmark equilibrium, no firm can profit from shrouding its add-on prices or from charging different prices. Hence, there always exists an equilibrium in which all firms advertise their add-on prices and in which only the w-product is traded at the prices indicated in Proposition 1.

Does the presence of boundedly rational consumers create scope for the trade of inferior products? Boundedly rational consumers may not understand that the w-product offers a higher surplus to them than the v-product. Consider the following setting: All firms charge the same prices and shroud add-on prices; they charge a positive base price for the w-product as well as a zero base price for the v-product; add-on prices are p^w_{add} and p^v_{add} for w-product and v-product, respectively; the total prices for the two products are such that $w - p^w > v - p^v$.

Rational consumers then anticipate the payoff from each good and purchase the w-product. In contrast, boundedly rational consumers may not correctly infer the total prices of both products. Suppose they purchase the v-product in equilibrium. The high price-state then occurs very frequently for them. They do not take into account that the frequency of the high price-state varies in the product type. Hence, boundedly rational consumers believe that the add-on price of the w-product is p^v_{add} instead of p^w_{add}. Since the base price of the w-product is positive, we have for each firm i that

$$v - p^v_{add} > w - p^w_{i,base} - p^v_{add}. \tag{7}$$

Hence, the v-product appears to boundedly rational consumers as more attractive than the w-product. For these consumers, perceived add-on prices are identical for both products, regardless of their equilibrium strategy. Purchasing the v-product is therefore the only personal equilibrium for boundedly rational consumers in the considered setting. We analyze under what circumstances there exists an equilibrium that features such an outcome.

Proposition 2 (Trade of Inferior Products)**.** *Suppose there is a share of boundedly rational consumers and that the maximal add-on price is small enough such that* $v - p^v_{add} \geq \frac{1}{3}(v - c^v)$.

(i) *If* $\frac{t}{n} < p^v_{add} - c^v$, *there exists a symmetric shrouding equilibrium in which rational consumers purchase the w-product at price* $p^w = c^w + \frac{t}{n}$ *and boundedly rational consumers purchase the v-product at price* $p^v = p^v_{add}$.

(ii) *If* $p^v_{add} - c^v \leq \frac{t}{n} \leq \frac{2}{3}(v - c^v)$ *and* $\tilde{p} > (w - c^w) - (v - c^v)$, *there exists a symmetric shrouding equilibrium in which rational consumers purchase the w-product at price* $p^w = c^w + \frac{t}{n}$ *and boundedly rational consumers purchase the v-product at price* $p^v = c^v + \frac{t}{n}$.

(iii) *If* $\frac{2}{3}(v - c^v) < \frac{t}{n} \leq \frac{2}{3}(w - c^w)$, *there exists no equilibrium in which the v-product is traded.*

(iv) *If* $\frac{2}{3}(w - c^w) < \frac{t}{n} \leq w - c^w$, *there exists no symmetric equilibrium in which the v-product is traded. In any equilibrium that maximizes industry profits, firms only trade the w-product at price* $p^w = w - \frac{t}{2n}$.

(v) *If* $w - c^w < \frac{t}{n}$, *there exists no equilibrium in which the v-product is traded.*

The proof of Proposition 2 is in Appendix A.2. The qualification $v - p^v_{add} \geq \frac{1}{3}(v - c^v)$ is not essential for the result, but it saves us from further case distinctions. Figure 3 below illustrates Proposition 2 by showing the profits that firms earn in the symmetric (shrouding) equilibrium.

Consider first Case (i) where the competition between firms is intense enough so that $\frac{t}{n} < p^v_{add} - c^v$. Then there exists a shrouding equilibrium with the features described above. Firms sell the w-product to rational consumers at the competitive price $p^w = c^w + \frac{t}{n}$. To boundedly rational consumers they sell the v-product at the total price p^v_{add}; its base price is zero and the add-on price is maximal. Each group of consumers is convinced to purchase the best deal in the market, but only for rational consumers this is actually true. An individual firm's profit in the symmetric shrouding equilibrium equals

$$\pi^{sh} = \frac{1}{n}\left[\lambda(p^v_{add} - c^v) + (1 - \lambda)\frac{t}{n}\right]. \tag{8}$$

The firms' strategies support an equilibrium due to the logic outlined in HKM17: Firms earn higher profits from boundedly rational consumers than from rational consumers (in Figure 3, the difference in profit levels is displayed by the black solid and the red solid line). It does not pay off for firms to educate consumers in order to sell them the superior w-product since this product can only be sold at relatively low prices.

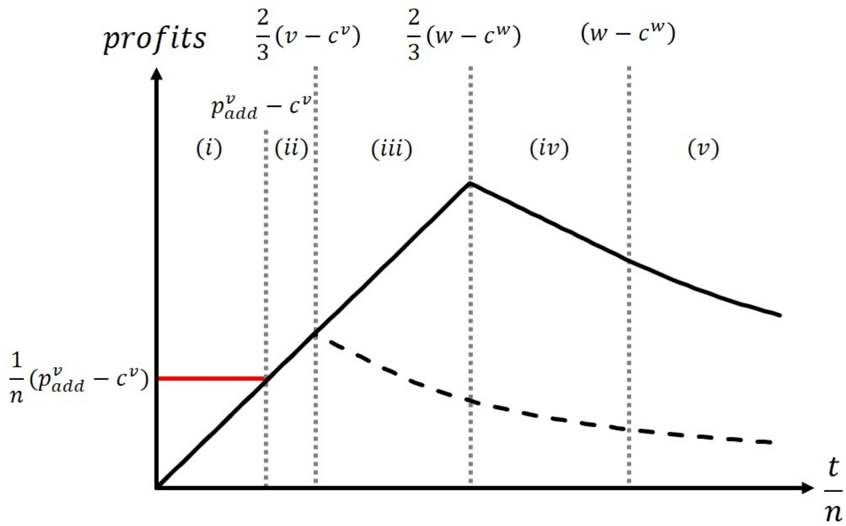

Figure 3: Individual firm profits from trading the v-product in the symmetric shrouding equilibrium (red line), individual firm profits from trading the w-product in the symmetric shrouding and in the symmetric benchmark equilibrium (sold black line), and individual firm profits from selling the v-product in a symmetric benchmark equilibrium when only the v-product is available (dashed black line). The numbers (i) to (v) indicate the cases from Proposition 2.

Next, consider Case (ii) where transport costs are such that $p^v_{add} - c^v \leq \frac{t}{n} \leq \frac{2}{3}(v - c^v)$. In this region, there still can exist a symmetric equilibrium in which firms sell then inferior product to boundedly rational consumers. However, they no longer benefit from this relative to the benchmark equilibrium outcome. The equilibrium markup on both the w- and the v-product equals $\frac{t}{n}$. To charge this markup on the v-product, firms have to choose both a positive base price and the maximal add-on price. That is, competitive pressure is no longer enough to reduce the base price of the v-product to zero. Moreover, the difference $\tilde{p} = p^v_{add} - p^w_{add}$ must be large enough so that the v-product still appears as the superior deal to boundedly rational consumers. To see this, note that they strictly prefer the v-product to the w-product at the symmetric equilibrium prices if and only if

$$v - \left(c^v + \frac{t}{n} - p^v_{add}\right) - p^v_{add} > w - \left(c^w + \frac{t}{n} - p^w_{add}\right) - p^v_{add}, \tag{9}$$

which is equivalent to $\tilde{p} > (w - c^w) - (v - c^v)$. Overall, the only effect of trade of inferior products in this domain is that gains from trade are wasted. There is no longer a strong case for the trade of inferior products.

Finally, consider the Cases (iii) to (v). Transport costs satisfy $\frac{t}{n} > \frac{2}{3}(v - c^v)$ in these domains. There no longer exists a symmetric equilibrium in which the inferior product is sold to boundedly rational consumers.[7] The reason is that it no longer pays off for firms to sacrifice gains from trade by trading the inferior product as they extract a large share of these gains. If firms would shroud add-on prices and some consumers would purchase the v-product, then at least one firm could profit from educating consumers and offering only the w-product.

The significance of Proposition 2 is that it shows when the shrouding equilibrium from the multi-product version of HKM17 also obtains in an environment where all consumers correctly predict equilibrium outcomes. Taking the Cases (i) to (v) together implies that this happens only if competition is sufficiently intense. In our version of the shrouding equilibrium, boundedly rational consumers do not learn anything on the equilibrium path that would inform them about the misspecification in their reasoning. Therefore, shrouding equilibria with inefficient trade can persist even when consumers and firms interact frequently.

From Proposition 2 we obtain clear welfare implications. When firms strictly profit from shrouding add-on prices – in the domain of Case (i) – the surplus of boundedly rational consumers is reduced by $(v-p^v_{add})-(w-c^w-\frac{t}{n})$, and total welfare is reduced by $\lambda[(w-c^w)-(v-c^v)]$ relative to the benchmark equilibrium. A policy that could increase the surplus of boundedly rational consumers is add-on price regulation, i.e., in our framework, the limit $\bar{p}$ on the second price that firms can charge. A reduction in $\bar{p}$ implies a reduction of the maximal add-on price p^v_{add} and hence a reduction in the price that boundedly rational consumers pay for the inferior product. Proposition 2 implies that a reduction in the maximal add-on price p^v_{add} has two effects. First, if the shrouding equilibrium survives the intervention, it redistributes surplus from firms back to boundedly rational consumers. Second, the intervention may render shrouding add-on prices unprofitable for firms when the new maximal add-on price p^v_{add} is small enough such that $\frac{t}{n} \geq p^v_{add} - c^v$. Firms then earn the same markup from both w- and v-product. Thus, keeping maximal add-on prices low may prevent the introduction and trade of inferior products.

3.3 Optimal Add-on Prices

When firms strictly profit from shrouding add-on prices, the total price of the v-product equals the maximal add-on price p^v_{add}. Their profit in a shrouding equilibrium therefore depends on p^v_{add}. In the following, we characterize the maximal add-on price for the v-product that maximizes industry profits and call it the "optimal add-on price." In the next subsection, we examine a mechanism that potentially establishes the optimal add-on price in equilibrium.

[7] For the case of intermediate transport costs, $\frac{2}{3}(w-c^w) < \frac{t}{n} \leq w-c^w$, we cannot completely rule out that there is an asymmetric equilibrium in which at least some firms sell the v-product to boundedly rational consumers. However, such an equilibrium is not optimal for firms. In this region, the equilibrium outcome that maximizes industry profits is to only trade the w-product at the prices from the symmetric benchmark equilibrium. Thus, there is little reason to offer the v-product.

We consider first the case when competition is sufficiently relaxed so that $\frac{t}{n} > \frac{2}{3}(v - c^v)$. In this domain, the industry profit from selling the w-product in the benchmark equilibrium is strictly larger than the maximal industry profit from selling the v-product; one can see this directly from Figure 3. This holds for any value of the maximal add-on price p^v_{add}. Hence, for maximal industry profits, the maximal add-on price is irrelevant in this domain.

Next, we assume that competition is intense enough such that $\frac{t}{n} \leq \frac{2}{3}(v - c^v)$. We examine which total price p^v for the v-product would firms choose in a cartel to maximize the industry profit from trading with boundedly rational consumers. This price would be chosen such that the marginal boundedly rational consumers are indifferent between trading (with either of the neighboring firms) and not trading at all, i.e., $v - p^v - \frac{t}{2n} = 0$. Firms charge this total price in a symmetric shrouding equilibrium only if the maximal add-on price takes on exactly this value. As in the previous subsection, we can show that there exists a symmetric shrouding equilibrium in which firms sell the v-product at total price p^v to boundedly rational consumers, and the w-product at the symmetric benchmark equilibrium price $c^w + \frac{t}{n}$ to rational consumers. From this, we get the following result.

Corollary 1 (Optimal Add-On Price)**.** *If $\frac{t}{n} \leq \frac{2}{3}(v - c^v)$, firms earn the maximal equilibrium industry profit in a symmetric shrouding equilibrium when the maximal add-on price for the v-product equals*

$$p^v_{add}(t, n) = v - \frac{t}{2n}.$$

If $\frac{t}{n} > \frac{2}{3}(v - c^v)$, then in the equilibrium that maximizes the industry profit firms only sell the w-product to consumers so that the maximal add-on price p^v_{add} does not affect their profits.

Figure 4 shows a firm's profit in the symmetric (shrouding) equilibrium when for each level of competition $\frac{t}{n}$ the optimal add-on price $p^v_{add}(t, n)$ is implemented. The red line displays the profit a firm makes from selling the v-product to boundedly rational consumers, and the black line displays a firm's profit from selling the w-product to any consumer type. Under the optimal add-on price, a firm's profit in the symmetric shrouding equilibrium equals

$$\tilde{\pi}^{sh} = \frac{1}{n}\left[\lambda(v - \frac{t}{2n} - c^v) + (1 - \lambda)\frac{t}{n}\right]. \tag{10}$$

As the market becomes more competitive, the profits from selling the inferior product increase, while the profits from selling the superior product decrease. This reflects the differential price setting for the two products: The optimal add-on price $p^v_{add}(t, n)$ is set to extract the maximal joint profit from selling the v-product like in a price cartel. As $\frac{t}{n}$ decreases, consumers become more homogeneous so that the cartel price also increases. In contrast, the price for the superior w-product is determined by competition between firms. Therefore, the firms' markup on this product (and hence its price) becomes smaller and smaller as $\frac{t}{n}$ decreases. At $\frac{t}{n} = 0$ the rational consumers' surplus equals $w - c^w$, while boundedly rational consumers earn no surplus at all.

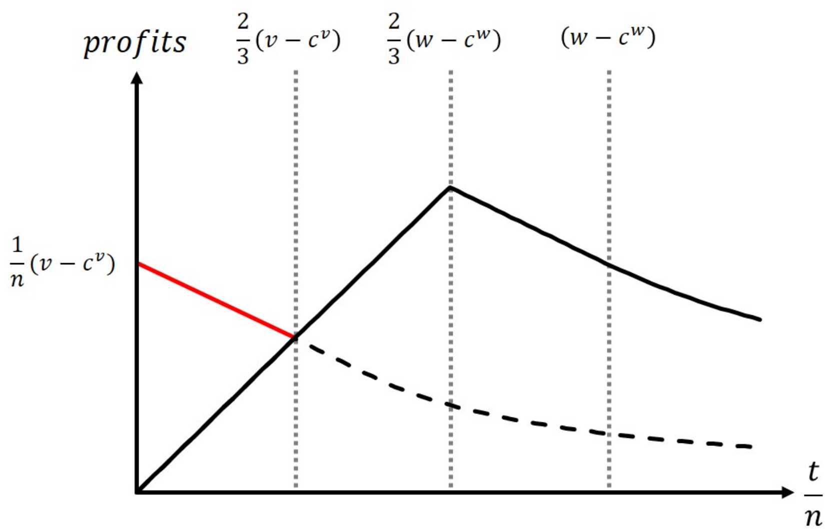

Figure 4: Individual firm profits from trading the v-product in the symmetric shrouding equilibrium when the optimal add-on price is implemented (red line); all other details as in Figure 3.

The symmetric shrouding equilibrium under the optimal add-on price has two further features. First, almost all boundedly rational consumers would only trade with the closest firm ("their firm"). That is, they would prefer no trade to trade with any other firm. Second, if the market is sufficiently competitive, then boundedly rational consumers would also prefer no trade to purchasing the w-product from any firm. To see this, note that at $\frac{t}{n} \approx 0$ we have $w - p^v_{add}(t, n) \approx w - v < 0$. Therefore, the extent to which boundedly rational consumers overestimate the total price of the superior product increases in the degree of competition.

3.4 Exploitative Innovation

Firms may be able to affect the maximal add-on price, for example, by creating new contract terms and add-on charges, or by inventing new ways to circumvent industry regulations. Such practices are often easy to copy. Once a firm establishes a new practice profitably, others can follow suit quickly. HKM16 call such developments "exploitative innovation." In this subsection, we adapt their model of exploitative innovation to our framework and show that such innovation may establish the optimal add-on price as the maximal add-on price in equilibrium.

We take the model from Section 2 and add an additional stage. Before firms set their prices and choose their advertising strategies, there is an "innovation stage" where one firm, say firm 1, can invest into an exploitative innovation project. Only one project is available and after its completion, the innovation is available to all firms. We therefore focus on non-appropriable innovation.[8] Exploitative innovation increases the maximal add-on price for the v-product from p^v_{add} to $\tilde{p}^v_{add}$.[9] After the innovation stage, the game continues with the realized level of the maximal add-on price. Denote by $I_{p^v_{add}}$ the investment that firm 1 is willing to make to realize exploitative innovation, and $\Delta p^v_{add} = \tilde{p}^v_{add} - p^v_{add}$. We then obtain the following result.

Corollary 2 (Exploitative Innovation)**.** *Consider the exploitative innovation extension of our model. Suppose that $\frac{t}{n} \leq \frac{2}{3}(v - c^v)$ and that the continuation equilibrium after the innovation stage is the symmetric shrouding equilibrium. If the maximal add-on prices $p^v_{add}, \tilde{p}^v_{add}$ satisfy $p^v_{add} < \tilde{p}^v_{add} \leq p^v_{add}(t, n)$, firm* 1 *is willing to invest*

$$I_{p^v_{add}} = \frac{\lambda}{n} \max\left\{0, \tilde{p}^v_{add} - \max\left\{p^v_{add}, c^v + \frac{t}{n}\right\}\right\}$$

into exploitative innovation; for sufficiently small $\frac{t}{n}$ this value equals $I_{p^v_{add}} = \frac{\lambda}{n}\Delta p^v_{add}$. However, if $p^v_{add} \geq p^v_{add}(t, n)$, we have $I_{p^v_{add}} \leq 0$.

When firms benefit in equilibrium from shrouding add-on prices, there are incentives for exploitative innovation as in HKM16. However, in contrast to their setting, the scope for exploitative innovation is limited in our framework, due to the fact that boundedly rational consumers correctly anticipate the total price in equilibrium. Firm 1 is willing to invest into raising p^v_{add} as long as this value does not exceed the optimal add-on price $p^v_{add}(t, n)$. Beyond this threshold, the willingness to pay for exploitative innovation is negative since firms then would lose boundedly rational consumers and earn lower profits after the innovation. Therefore, if firms can continuously invest into exploitative innovation, the optimal add-on price $p^v_{add}(t, n)$ would be the limit of their efforts.

3.5 Product and Process Innovation

An important question in industrial organization is whether firms have incentives to invest into innovation and new technologies. In particular, the trade of inferior products in a shrouding equilibrium may affects firms' innovation incentives. HKM16 obtain a negative result in this respect: While firms always have incentives to invest into exploitative innovation, they have no incentives to invest into product innovation which would increase the consumers' utility, provided that innovation is non-appropriable. In this subsection, we apply the model of non-appropriable innovation from HKM16 to our multi-product framework. In contrast to HKM16,

[8]In their main result (Proposition 2), HKM16 also focus on non-appropriable innovation. We think that for our main application – financial services – this is the empirically relevant case for exploitative innovation.

[9]This means that it increases the maximal second price $\bar{p}$. We can safely ignore the corresponding effect on p^w_{add} as this value does not affect the total price of the w-product.

we find that, in a shrouding equilibrium with inefficient trade, firms have incentives to innovate. These incentives constitute a self-correcting force that reduces the welfare loss from inefficient trade or even reverses the welfare implications of hidden add-on prices. To cleanly disentangle the different forces, we first consider product innovation, then process innovation, and then allow for general innovation projects that can make the v-product the superior product.

Product Innovation. We again assume that there is an innovation stage where firm 1 can invest into an innovation project before the market opens. This innovation is then available to all firms. Specifically, firm 1 can invest into increasing the utility each product: w-product innovation (v-product innovation) increases the utility of the w-product (v-product) from w to $\tilde{w} > w$ (from v to $\tilde{v} > v$). Denote by I_w and I_v the investment that firm 1 is willing to make to realize the w-product and v-product innovation, respectively. Define $\Delta v = \tilde{v} - v$. We first assume that the v-product is the inferior product even after v-product has been conducted. The following result provides an overview under what circumstances firm 1 would invest into product innovation.

Corollary 3 (Product Innovation)**.** *Consider the product innovation extension of our model. Suppose that* $\frac{t}{n} \leq \frac{2}{3}(v - c^v)$ *and that the continuation equilibrium after the innovation stage is the symmetric shrouding equilibrium. If all consumers are rational, then firm* 1 *does not invest into product innovation,* $I_w = I_v = 0$. *If there is a share of boundedly rational consumers, then the following statements hold:*

(i) Firm 1 *does not invest into w-product innovation,* $I_w = 0$.

(ii) Assume that the maximal add-on price p^v_{add} *is given. Firm* 1 *then does not invest into v-product innovation,* $I_v = 0$.

(iii) Assume that maximal add-on price always equals the optimal level $p^v_{add}(t, n)$ *in the continuation equilibrium after the innovation stage. Firm* 1 *is then willing to invest* $I_v = \frac{\lambda}{n}\Delta v$ *into v-product innovation.*

The restriction $\frac{t}{n} \leq \frac{2}{3}(v - c^v)$ ensures that we only consider interesting cases where the benchmark equilibrium is competitive and where a symmetric shrouding equilibrium with inefficient trade can exist. First, observe that if all consumers are rational, then there are no innovation incentives in the benchmark equilibrium. In this equilibrium, firms only sell the w-product to consumers at price $p^w = c^w + \frac{t}{n}$, and earn profit $\frac{t}{n^2}$. Thus, firm profits are independent of the value of the w-product. Firm 1 has no incentive to invest into w-product innovation since any gain from innovation would be passed on to the consumers.

Next, Corollary 3 considers the case when there are boundedly rational consumers and the continuation equilibrium after the innovation stage is the symmetric shrouding equilibrium. All statements on this case directly follow from the profit functions in equation (8) and (10). Again, there are no incentives to invest into w-product innovation. As in the benchmark equilibrium, the markup on this product is $\frac{t}{n}$, regardless of its value to consumers. Similarly, firm 1 has no incentive to invest into v-product innovation as long as the maximal add-on price p^v_{add} is fixed. Since it cannot increase the price for the product, only boundedly consumers would benefit from an improvement of the inferior product.

This changes when the add-on price can be kept at the optimal level $p^v_{add}(t,n)$ through continuous exploitative innovation. Firm 1 is then willing to invest into v-product innovation. Intuitively, the combination of exploitative and product-innovation means that firm 1 offers more value to boundedly rational consumers and reaps the additional gains from trade by adjusting the price in a way so that competition does not threaten the increase in profits. Statements (ii) and (iii) in Corollary 3 imply that add-on price regulation may involve a trade-off between welfare and consumer surplus. Tighter limits on add-on prices redistribute surplus from firms to boundedly rational consumers. However, they also reduce firms' incentive to invest into v-product innovation, which would reduce the welfare loss from the trade of inferior products.

Process Innovation. We next consider the case where firm 1 can invest into an innovation project that reduces production costs. Unlike exploitative and product innovation, process innovation is not considered in HKM16, but it can be analyzed in the same fashion. We allow for w-process and v-process innovation: w-process innovation (v-process innovation) decreases the cost of the w-product (v-product) from c^w to $\tilde{c}^w < c^w$ (from c^v to $\tilde{c}^v < c^v$). Again, we assume that the v-product is the inferior product even after v-product or v-process innovation has been conducted. Denote by I_{c^w} and I_{c^v} the amount that firm 1 is willing to invest into w-product and v-product innovation, respectively. We obtain the following result.

Corollary 4 (Process Innovation). *Consider the process innovation extension of our model. Suppose that $\frac{t}{n} \leq \frac{2}{3}(v - c^v)$ and that the continuation equilibrium after the innovation stage is the symmetric shrouding equilibrium. If all consumers are rational, then firm* 1 *does not invest into process innovation, $I_{c^w} = I_{c^v} = 0$. If there is a share of boundedly rational consumers, then the following statements hold:*

(i) Firm 1 *does not invest into w-process innovation, $I_{c^w} = 0$.*

(ii) Firm 1 *is willing to invest*

$$I_{c^v} = \frac{\lambda}{n} \max\left\{0, (p^v_{add} - \tilde{c}^v) - \max\left\{p^v_{add} - c^v, \frac{t}{n}\right\}\right\}$$

into v-process innovation; for $\frac{t}{n}$ close enough to zero this value equals $I_{c^v} = \frac{\lambda}{n}\Delta c^v$.

As for the case of product innovation, there are no incentives to invest into process innovation if a product is traded at the competitive price. Thus, there are no investments into process innovation in the benchmark equilibrium, and no investments into w-process innovation in the symmetric shrouding equilibrium. However, in the symmetric shrouding equilibrium, firms' profits decrease in the production costs of the v-product. One can see this directly from equation (8). Hence, firm 1 is willing to invest into v-process innovation. We therefore again obtain a self-correcting force that reduces the welfare loss created through inefficient trade in a shrouding equilibrium. Unlike the other results in this paper, the findings on process innovation are *not* driven by the equilibrium beliefs of boundedly rational consumers. Welfare-enhancing process innovation would also occur in the HKM16 setting (but is not mentioned in that paper).

General Innovation Projects. Product and process innovation reduce the welfare-loss from the trade of inefficient products. To what extent do innovation incentives persist if the welfare-loss is reduced to zero and the v-product becomes the superior product? In the following, we consider the general case and allow that the v-product offers more gains from trade than the w-product after innovation has taken place. We consider v-innovation that increases the value of the v-product from v to $\tilde{v} > v$ and decreases its production costs from c^v to $\tilde{c}^v < c^v$. Accordingly, we define w-innovation. The maximal add-on price always equals the optimal level $p^v_{add}(t, n)$ in the continuation equilibrium after the innovation stage. The continuation equilibrium is symmetric and equals the symmetric shrouding equilibrium whenever it exists.

We first verify that a symmetric shrouding equilibrium may exist even if the v-product becomes the superior product. In this equilibrium, all firms charge $p^w = c^w + \frac{t}{n}$ for the w-product and $p^v = \tilde{v} - \frac{t}{2n}$ for the v-product (with a first price of zero and the second price at the optimal level). We can verify that, at these prices, rational consumers strictly prefer the w-product and boundedly rational consumers strictly prefer the v-product as $\tilde{v} > w$. Firms cannot deviate profitably without advertising add-on prices. Hence, it remains to check whether a deviation that involves advertising add-on prices can be profitable. The next result shows that this is not the case if $\tilde{v} - \tilde{c}^v$ is not too large relative to $w - c^w$.

Proposition 3 (General Innovation Projects)**.** *Consider the general innovation extension of our model. Suppose that $\frac{t}{n} \leq \frac{2}{3}(v - c^v)$ and that the continuation equilibrium after the innovation stage is symmetric and equals the symmetric shrouding equilibrium whenever it exists. If all consumers are rational, then firm* 1 *does not invest into innovation. If there is a share of boundedly rational consumers, then the following statements hold:*

(i) Firm 1 *does not invest into w-innovation.*

(ii) There is a value $G^{max}(\lambda, t, n) > w - c^w$ so that firm 1 is willing to invest a positive amount into v-innovation if and only if $\tilde{v} - \tilde{c}^v \leq G^{max}(\lambda, t, n)$. The value $G^{max}(\lambda, t, n)$ strictly increases in λ, and for $\frac{t}{n} \approx 0$ we have $G^{max}(\lambda, t, n) \approx \frac{n}{n-\lambda}(w - c^w)$.

We can draw two important conclusions from Proposition 3. First, the presence of boundedly rational consumers may provide innovation incentives that go beyond the elimination of welfare-losses due to inefficient trade. It can therefore lead to more welfare than in a competitive market with only rational consumers. The intuition follows Schumpeter's (1943) proposed link between market structure and R&D investments. Boundedly rational consumers create the market power necessary to make investments into innovation profitable. In particular, this holds for non-appropriable innovation. The presence of boundedly rational consumers works like patent protection for v-innovation. Second, innovation incentives for the v-product are not unlimited. If the gains from trade from the v-product are too large relative to those from the w-product, this creates an incentive to educate all consumers and to serve a larger fraction of rational and boundedly rational consumers than in the symmetric shrouding equilibrium. This incentive increases in the number n of firms and decreases in the share λ of boundedly rational consumers. If the symmetric shrouding equilibrium is no longer sustainable, then in equilibrium firms charge the competitive markup $\frac{t}{n}$. Therefore, firm 1 refrains from innovation that destroys the symmetric shrouding equilibrium.

Overall, our results on innovation incentives provide a much more positive perspective on markets with shrouded add-on prices than the previous literature. There are incentives for exploitative innovation, but their scope is limited since consumers correctly anticipate equilibrium outcomes. Moreover, continuous exploitative innovation creates incentives for firms to increase the value of the inferior product, which reduces the welfare loss that is due to its trade. Additionally, firms have incentives to reduce the production costs of the inferior product. These innovation incentives not only reduce the welfare-loss from inefficient trade, but may even lead to a better market outcome than if all consumers were rational.

3.6 The Model without Education

In our baseline model, we assumed that firms can educate consumers free of charge by advertising add-on prices. This assumption is made in the hidden add-on price literature to demonstrate the robustness of shrouding equilibria, that is, to show that they can exist even if education is costless. Our main result in Proposition 2 shows when shrouding equilibria with inefficient trade exist and when this is not the case. In the proofs of the statements (iii) to (v) of Proposition 2, we explicitly use the assumption of free consumer education. However, this is an extreme assumption that is most likely violated in applications. As we discuss in Section

5, providing information on add-on prices often has very little impact on behavior. It may be very costly to fundamentally change consumers' understanding of complex pricing schedules. This raises the question to what extent our results also would obtain when it is impossible or prohibitively costly to educate consumers.

To answer this question, we consider in Appendix A.3 the model from Section 2, but assume that firms cannot advertise add-on prices to educate consumers. Firms only set prices and the behavior of boundedly rational consumers must be a personal equilibrium according to Definition 1. We analyze the set of equilibria in this new game and obtain the following results: If the market is sufficiently competitive, $\frac{t}{n} < v - p^v_{add}$, then in the unique symmetric equilibrium firms sell the w-product to rational consumers at price $p^w = c^w + \frac{t}{n}$, and the v-product to boundedly rational consumers at price $p^v = p^v_{add}$ (as in the shrouding equilibrium of Proposition 2). If competition is sufficiently relaxed, $\sqrt{\frac{1}{2}(v - c^v)} \leq \frac{t}{n}$, then in a profit-maximizing equilibrium firms only sell the w-product to consumers (other equilibria may exist though). For intermediate levels of competition, there exists a unique symmetric equilibrium in which both products are sold to consumers.

4 Shrouded Add-on Prices and Substitution Effort

The canonical model of hidden add-on pricing with myopic consumers is GL06. Their framework has been used and extended in several papers.[10] We therefore devote a chapter to it. In their model, firms offer a single product that comes with an add-on. Firms choose prices for the product and the add-on, and decide whether to advertise or shroud the add-on price. There are sophisticated and myopic consumers. Sophisticated consumers anticipate the add-on price and substitute away from the add-on when the expected add-on price exceeds the costs of substitution effort. Myopic consumers ignore the fact that they have to pay for the add-on price if they do not exert substitution effort. If at least one firm advertises its add-on price, myopic consumers become sophisticated. A shrouding equilibrium exits if the share of myopic consumers is sufficiently large. This equilibrium implies a transfer of wealth from myopic to rational consumers. It is inefficient since rational consumers exert substitution effort, while firms can provide the add-on without incurring additional costs.

In this section, we first replicate the original result from GL06 with boundedly rational consumers who correctly anticipate their equilibrium expenses. The main result in GL06 therefore does not rely on the assumption of consumer myopia. Next, we show that a shrouding equilibrium again only exists if the market is sufficiently competitive. This somewhat contrasts with

[10] In particular, Armstrong and Vickers (2012), Dahrenmöller (2013), Ko and Williams (2017), Kosfeld and Schüwer (2017), and Johnen (2020) directly build on the GL06 framework.

the original version of the result, and we will explain in detail what causes this difference. Finally, we discuss the implications of our version of the GL06 result for dynamic competition. All proofs for this section can be found in Appendix A.4.

4.1 Setup

We adapt the GL06 model to our framework. Firms offer only the v-product and the unit cost of this product equals c. Consumers choose whether to purchase a v-product and whether they exert substitution effort or not. Let $a = 2$ represent purchasing a v-product and exerting effort, $a = 1$ purchasing a v-product and exerting no effort, and $a = 0$ purchasing no product and exerting no effort. Substitution effort creates personal costs of e for the consumer, but reduces the probability of the high price-state. The probability distribution over all variables is as in the basic framework described in Section 2, and the subjective causal model of boundedly rational consumers is again given by $\mathcal{R}$, the graph on the right in Figure 1. Hence, boundedly rational consumers do not understand that exerting substitution effort would change the distribution over price-states. Since it is costly, they see no value in exerting effort.

Each firm i choses a first price $p_{i,1}$ and a second price $p_{i,2}$. As in the original model, we assume that the base price can take on all values in $\mathbb{R}$. The maximal second price is $\bar{p} > 0$. When the low price-state realizes, a consumer who purchased the v-product from firm i pays $p_{i,1}$. If the high price-state realizes, she pays $p_{i,1} + p_{i,2}$. The base price of firm i's v-product equals

$$p_{i,base} = p(x_3 = 1 \mid a = 2)\, p_{i,1} + p(x_3 = 2 \mid a = 2)\, (p_{i,1} + p_{i,2}), \tag{11}$$

and the add-on price of firm i's v-product is given by

$$p_{i,add} = [p(x_3 = 2 \mid a = 1) - p(x_3 = 2 \mid a = 2)]\, p_{i,2}. \tag{12}$$

Thus, the base price $p_{i,base}$ is the expected price a consumer pays if she trades with firm i and exerts substitution effort, and $p_{i,base} + p_{i,add}$ is the expected price she pays if she trades with firm i and does not exert substitution effort. The maximal add-on price is given by

$$p_{add} = [p(x_3 = 2 \mid a = 1) - p(x_3 = 2 \mid a = 2)]\, \bar{p}. \tag{13}$$

We assume that p_{add} is strictly larger than the costs of substitution effort e. As in the original model, firms can either shroud or advertise their add-on prices. If all firms shroud their add-on prices, boundedly rational consumers remain boundedly rational. If at least one firm advertises its add-on price, all boundedly rational consumers become rational. The rest of the model (sequence of events, equilibrium definition) is the same as in Section 2.

4.2 Shrouding Equilibria

To examine the existence and features of shrouding equilibria, we proceed in two steps. First, we assume that firms cannot advertise add-on prices and describe the symmetric equilibrium outcome for this case. Second, we consider the full model where firms can advertise add-on prices and analyze under what circumstances the equilibrium outcome from the first step is indeed an equilibrium outcome in the full model.

We begin with the first step. If firms do not advertise add-on prices, all consumers anticipate that in the high price-state they have to pay the maximal second price $\bar{p}$. To reduce the likelihood of these additional charges, rational consumers exert substitution effort and therefore only pay, in expectation, the base price $p_{i,base}$ when they trade with firm i. Boundedly rational consumers do not understand that substitution effort would help them to reduce the likelihood of the high price-state. Since substitution effort is costly, they are not performing it and end up paying both the base and the maximal add-on price, $p_{i,base} + p_{add}$, when they trade with firm i. Given this consumer behavior, the symmetric equilibrium outcome is as follows.

Lemma 1 (Symmetric Shrouding Equilibrium Outcome)**.** *Consider the model with substitution effort of this section. Suppose that firms cannot advertise their add-on prices, and that the maximal add-on price is small enough such that $v - c \geq 3p_{add}$. The unique symmetric equilibrium outcome is then as follows:*

(i) If $\frac{t}{n} \leq \frac{2}{3}[(v-c) - (1-\lambda)p_{add}]$, firms serve all consumers and charge the base price

$$p_{base} = c - \lambda p_{add} + \frac{t}{n}.$$

(ii) If $\frac{2}{3}[(v-c)-(1-\lambda)p_{add}] < \frac{t}{n} \leq \frac{2}{3+\lambda}[(1+\lambda)(v-c)-(1-\lambda)p_{add}]$, firms serve all consumers, the marginal naive consumers are indifferent between trading and not trading, and firms charge the base price

$$p_{base} = v - p_{add} - \frac{t}{n}.$$

(iii) If $\frac{2}{3+\lambda}[(1+\lambda)(v-c) - (1-\lambda)p_{add}] < \frac{t}{n} \leq \frac{2}{3+\lambda}[(1+\lambda)(v-c) + 4\lambda p_{add} - (1+3\lambda)e]$, firms serve all rational consumers, the marginal boundedly rational consumers are indifferent between trading and not trading, and firms charge the base price

$$p_{base} = \frac{2\lambda(v + c - 2p_{add}) + (1-\lambda)(c + \frac{t}{n})}{1 + 3\lambda}.$$

(iv) If $\frac{2}{3+\lambda}[(1+\lambda)(v-c) + 4\lambda p_{add} - (1+3\lambda)e] < \frac{t}{n} \leq (v-c) + 2\lambda p_{add} - (1+\lambda)e$, firms serve all rational consumers, marginal rational and boundedly rational consumers are

indifferent between trading and not trading, and firms charge the base price

$$p_{base} = v - e - \frac{t}{n}.$$

(v) If $(v - c) + 2\lambda p_{add} - (1 + \lambda)e < \frac{t}{n}$, firms serve only a share of rational and boundedly rational consumers, and they charge the base price

$$p_{base} = \frac{v + c}{2} - \frac{2\lambda p_{add} + (1 - \lambda)e}{2}.$$

The restriction on p_{add} is not essential for Lemma 1. It only saves us from a few more case distinctions. In particular, Case (i) and Case (ii) hold for any value of p_{add}. At higher levels of transportation costs, it may no longer be profitable for firms to serve any boundedly rational consumers if p_{add} is very large (as doing so would require to set a very low base price). However, if $v - c \geq 3p_{add}$, firms serve at least some boundedly rational consumers at all levels of transportation costs, regardless of their share λ.

Lemma 1 describes which base price firms set at varying levels of competition in the symmetric equilibrium. Firms do not serve all consumers if transport costs are sufficiently large. In Lemma 1, this happens from Case (iii) onwards. Since boundedly rational consumers pay $p_{add} - e$ more than rational consumers, the share of boundedly rational consumers firms serve is strictly smaller than the share of rational consumers who purchase the v-product. This is different from the original result in GL06 where myopic consumers ignore the add-on price.

To what extent do shrouding equilibria survive if firms are allowed to educate consumers by unshrouding their add-on prices? In the original result, shrouding add-on prices survives competitive pressure if there are sufficiently many myopic consumers. Through unshrouding, these consumers would realize that they can get advantageous deals when they exert substitution effort. The unshrouding firm cannot profitably match these deals. Therefore, it does not pay off for firms to unshroud add-on prices. To study whether this logic applies in our framework as well, we consider the different cases in Lemma 1 and obtain the following result.

Proposition 4 (Equilibrium in the Model with Substitution Effort)**.** *Consider the model with substitution effort of this section.*

(i) If $\frac{t}{n} \leq \frac{2}{3}[(v-c)-p_{add}]$, there exists a symmetric shrouding equilibrium whenever $\lambda \geq \frac{e}{p_{add}}$.

(ii) If $\frac{t}{n} < (v - c) + p_{add} - e$, there exists a symmetric shrouding equilibrium whenever λ is sufficiently large.

(iii) If $(v - c) + 2\lambda p_{add} < \frac{t}{n}$, there exists no shrouding equilibrium.

Proposition 4 indicates under what circumstances the original result from GL06 obtains. The first statement highlights that this is the case if competition is fierce enough. In this case, if $\lambda \geq \frac{e}{p_{add}}$, there is a symmetric shrouding equilibrium in which boundedly rational consumers pay both the base and maximal add-on price, rational consumers only purchase the base good and exert substitution effort. The only important difference to the original result is that, in our framework, boundedly rational consumers understand that they have to pay the add-on price; they just do not understand how to avoid it.

For intermediate levels of competition – starting from Case (ii) in Lemma 1 – firms would like to charge different base prices to rational and boundedly rational consumers in a shrouding equilibrium. Since this is not possible, firms lose profits from the fact that the marginal rational and boundedly rational consumers are typically at different locations, which makes shrouding less profitable. Nevertheless, this disadvantage vanishes as the share of boundedly rational consumers λ approaches unity. Therefore, deterrence from unshrouding remains effective if $(v - c) + p_{add} - e > \frac{t}{n}$ and λ is large enough.

Finally, if competition is sufficiently relaxed, $\frac{t}{n} > (v - c) + 2\lambda p_{add}$, shrouding equilibria no longer exist. At these levels, firm charge prices so that they would not lose customers to rival firms after unshrouding add-on prices. It is then profitable to unshroud add-on prices in order to sell the add-on to rational consumers who otherwise would exert substitution effort. The intuition for this effect is similar to that for the main result in Proposition 2. When firms enjoy sufficient market power, they sell products in a way to maximize the gains from trade.

We briefly contrast this finding with the original result. GL06 also allow for varying degrees of market power, and they show that the symmetric shrouding equilibrium is consistent with all markup levels. However, to get this result, GL06 assume a demand function that depends only on the difference between the (perceived) surplus of a firm's product and the (perceived) surplus of the rivals' best alternative product. This demand function can be microfounded by assuming a random utility model (Anderson et al. 1992). The consequence of this formulation is that, at any degree of market power, firms are competing against each other. In contrast, a sufficient degree of market power implies in our framework that the marginal (rational and boundedly rational) consumers become indifferent between trading with the closest firm and not trading at all. Given that all consumers correctly anticipate equilibrium expenses, this creates pressure to maximize the gains from trade by advertising add-on prices.

4.3 Implications for Dynamic Competition

Several papers extend the GL06 framework to further study markets with myopic consumers. One important extension is dynamic competition when firms collect information about their customers' types, and can tailor later offers to this information. Johnen (2020) analyzes such a framework and shows that a firm's informational advantage against its rivals translates into monopoly power and positive profits even when firms compete in Bertrand manner. An informed firm can charge a high base price to rational and a relatively low base price to myopic consumers. A rival can poach both consumer groups only with a uniform price. Hence, an adverse selection problem arises: As long as it does not charge a low price, it only attracts the unprofitable rational consumers.

A crucial assumption in Johnen (2020) is that myopic consumers remain myopic. Only then is information about their type valuable for firms. In the GL06 framework, this means that myopic consumers do not learn about the existence of hidden add-on prices even though they experience "surprise" add-on price charges in each period. This assumption is not plausible for high-frequency products such as credit or debit cards. However, our model shows that it is not needed. The myopic consumers in the original GL06 framework and the boundedly rational consumers in the present model behave in the same way in a shrouding equilibrium as long as the maximal add-on price is not too large. Hence, it is possible to obtain the main results on dynamic competition from Johnen (2020) in a framework where consumers are not assumed to be ignorant about their experiences with surprise charges.

5 Empirical Examples

In this section, we discuss the empirical research on two markets that frequently feature as examples for hidden add-on pricing: the market for credit and debit cards as well as the market for mutual funds. Both markets provide an opportunity to differentiate the predictions of our model with boundedly rational equilibrium beliefs from the predictions of a myopic consumer model. We argue that the model with boundedly rational beliefs can explain some phenomena in these markets that are difficult to reconcile with rational decision making or myopic beliefs.

5.1 The Market for Credit and Debit Cards

The most common form to finance purchases or to withdraw cash is to use debit or credit cards. Debit cards draw on existing deposits at a bank. Credit cards allow to borrow limited amounts from the card issuer. Some services that debit and credit cards provide can be very costly. Debit card users pay overdraft fees if they spend more than they have on their account. These overdraft fees can be significantly larger than the value of the transaction that caused the overdraft. Credit cards charge monthly interest on the consumer's outstanding balance. Since this is an unsecured loan, interest rates on credit card debt are relatively high. Additionally, there are penalty fees like over-limit and late fees.

For consumers, the cheapest way to finance purchases is to use a debit card and to maintain a sufficient balance so that no overdraft fees are incurred. However, a substantial fraction of consumers does not choose this strategy and pays substantial amounts for interests and penalty fees. Stango and Zinman (2009) analyze a two-year administrative data set on debit and credit card use of relatively sophisticated consumers. These consumers spend substantial amounts on interests and fees. The median consumer pays 43 USD per month. Around 60 percent of all interest rates and fees could have been avoided by using available checking balances or other credit cards. For most individuals, the monthly interest payments (and the amount that could be saved) are stable over time. In terms of fees, a share of individuals pays stable fees over time, but for half of the sample monthly fee payments are negatively correlated over time. Thus, a share of consumers tries to improve financial decision making after incurring fees. Similarly, Gathergood et al. (2021) find in an administrative data set spanning two years of credit card use that a share of consumers quickly learn to avoid fees by choosing an appropriate contractual arrangement, while others regularly incur high fees.

The market for liquidity provides an opportunity to test the implications of a model with myopic beliefs versus the implications of the present model with boundedly rational equilibrium beliefs. If consumers have myopic beliefs, they do not take interest rates and penalty fees into account when choosing their financing strategy. If consumers have boundedly rational equilibrium beliefs, they correctly anticipate their average expenses for interest and penalty payments. An intervention that directs consumers' attention to interest payments and penalty fees should reduce the sum of expenses on these items under myopic beliefs, but not under boundedly rational equilibrium beliefs (unless the intervention changes the boundedly rational consumers' subjective model $\mathcal{R}$). The fee structure of credit and debit cards has frequently been cited as an exploitative business practice by regulators and academics alike. Therefore, a number of studies examined whether informational interventions improve consumer behavior. Table 1 provides a summery of the results published so far.

As can be seen from the table, the effects of information on behavior (credit repayment, product choice, overdrafting) are either small or not significantly different from zero. The studies with most observations, Agarwal et al. (2015) and Seira et al. (2017), find no significant effects effects. These findings are quite surprising given that positive effects were highly anticipated in the context of consumers' low financial literacy.[11] The most significant effects are reported in the study by Stango and Zinman (2014). They find that survey questions on overdraft fees temporarily reduce the probability of incurring a fee by around 3.7 percentage points on a base likelihood of 30 percent (there is also a long-term effect of 1.7 percentage

[11] They were also surprising since publications in academic journals often exhibit a publication bias that favors positive results (e.g., Della Vigna and Linos 2021).

points). However, even in their sample individual overdraft behavior otherwise remains stable over time. Importantly, those individuals who change their behavior do this by reducing spending, and not by choosing a different product or selecting different contract terms or keeping a higher balance. Overall, these empirical finding suggest that consumers take add-on prices into account, but nevertheless choose a boundedly rational financing strategy.

TABLE 1 – Overview of Information Interventions Credit/Debit Cards

Study (observations, sample, country, duration)	Intervention (method)	Effect on product use
Credit Cards		
Agarwal et al. (2015) (160,000,000 credit card users, US, 5 years)	suggestion of alternative payment strategy, through CARD act (diff-in-diff, consumer and small business credit cards)	no significant effect on repayments, share borrowers who adopt strategy to pay off debt $\leq$ 36 months increases by 0.4 percentage points from base of 5.3 percent
Seira et al. (2017) (167,190 borrowers, Mexico, 10 months)	(a) salient personal interest rate (b) personalized months to pay (c) overconfidence warning (all RCTs)	no significant effect on debt/account closures no significant effect on debt/account closures tiny negative effect on debt, no effect on account closures
Medina (2021) (26,069 customers from finance platform, Brazil, 9 months)	reminders via push notifications on upcoming payments (RCT)	drop of late-payment fees by 2.6 percentage points on a base likelihood of 29.1 percent (partially offset by increased overdrafting)
Debit Cards		
Stango and Zinman (2014) (7,448 consumer survey participants, US, 3 years)	survey questions on overdrafts (natural experiment)	reduction of overdafting by 3.7 percentage points on a base likelihood of 30 percent
Alan et al. (2018) (108,000 customers from one bank, Turkey, 4 months)	(a) SMS-promotion of reduced overdraft fees (RCT) (b) SMS-promotion of overdraft availability (RCT)	reduction of overdrafting by 1.2 percentage points on a base likelihood of 31 percent increase in overdrafting by 0.9 percentage points (no long-run effects in both cases)

Our model further suggests that tightening the limit on add-on prices would benefit boundedly rational consumers. An example for such a regulation is the 2009 Credit Card Accountability Responsibility and Disclosure (CARD) Act. This legislation limited, in various ways, the extent to which companies could charge over-limit and late fees. Indeed, Agarwal et al.

(2015) find that it reduced the amount of penalty fees payed by consumers by 1.6 percent of the borrowing volume. The effect was particularly pronounced among "high-risk" consumers who are relatively likely to pay such fees. Also, there was no reduction in the volume of credit. These findings are inconsistent with the assumption of a competitive market with rational consumers. However, they are they are consistent with a competitive market in which firms profit from charing (anticipated) add-on prices and a fraction of consumers do not understand how alternative behaviors would lower their expenses.

5.2 The Market for Mutual Funds

Mutual funds are either actively managed funds where a fund manager makes investment decisions, or index funds that passively follow some stock index. Index funds are typically less costly in terms of management fees and often outperform actively managed funds (e.g., Fama and French 2010). Mutual funds are sold either directly to retail investors or indirectly through brokers who additionally offer investment advise to their clients. Financial advisers usually steer their clients towards expensive actively managed funds, which leads to reduced investment returns (e.g., Mullainathan et al. 2012, Hoechle et al. 2018, Chalmers and Reuter 2020). From a purely rational perspective, investments into actively managed funds are difficult to explain. The myopic consumer argument is that some customers do not take management fees into account and therefore invest into the fund that is recommended by their advisor. According to our model with boundedly rational equilibrium beliefs, boundedly rational consumers take the fees of a recommended fund into account, but do not understand that better investment opportunities exist. In the following, we first show that our model captures well the price effects triggered by index fund market entry. Then we discuss to what extent it better explains some empirical patterns in the market for mutual funds than a model with myopic beliefs.

Price effects of index fund entry. Actively managed funds exist in some form since the 19th century, while index funds have been introduced only gradually since the 1970s. Sun (2021) analyzes the prices of mutual funds and investment flows in the different market segments when an index fund becomes available. She exploits the staggered timing of index fund market entry in the different equity categories. Her main findings on prices and investment flows are as follows: In response to an index fund entry, the management fees of actively managed funds decrease when they are sold directly, but increase by roughly the same amount when they are sold through financial advisors. Actively managed funds significantly lose market share among investors who invest directly. Financial advisors hardly recommend any index funds.

These developments are consistent with our differentiated products model. As long as only the inferior v-product (the actively managed fund) is available, all consumers have the same beliefs about its costs and payoffs, so that it is sold at the competitive price $p^v = c^v + \frac{t}{n}$. When the superior w-product (the index fund) is introduced and the market enters a profitable shrouding equilibrium, rational consumers switch to the cheaper w-product which trades at price $p^w = c^w + \frac{t}{n} < c^v + \frac{t}{n} = p^v$. Boundedly rational consumers stick to the v-product, but they now pay a higher price for it since $p^v_{add} > c^v + \frac{t}{n} = p^v$.

Fees and transparency regulation. Unfortunately, there are no data to what extent the clients of financial advisors take fees into account when making investment decisions. However, the existence of management fees is most likely not a secret even to consumers with little experience. In their audit study, Mullainathan et al. (2012) find that many advisers mention fees in the discussion with the client, even when the "client" (the auditor) has not (yet) asked about them. Typically, they then argue that the product is worth paying these fees. This fits the setting of the present model where boundedly rational consumers anticipate the total price of the product they are purchasing, but not the total price of alternative products.

Further, as in the market for credit/debit cards, public policy tries to increase consumer surplus in the market for mutual funds by improving transparency. Since January 2018, the European Union imposes substantial requirements on firms that engage in financial advisory through an updated "Markets in Financial Instruments Directive" (Directive 2014/65/EU, henceforth MiFID II). MiFID II implements multiple regulations regarding the business model of financial advice, compensation schemes, product governance, and transparency. With respect to fees, it forces firms to disclose all costs of a product before the consumer purchases it, and, in regular intervals, also during the contractual relationship (Article 24(4)(c) of MiFID II). Thus, even when a consumer does not diggest all material that firms must provide due to MiFID II, the existence of fees is made very salient to her.

The effect of MiFID II on consumer behavior has not been analyzed yet and there exist few empirical studies on the general impact of the regulation. Loonen (2021) conducts a survey with 267 Dutch investment advisors. In particular, he asks about how advisors evaluate the effect of different MiFID II measures on investor protection. Cost transparency is evaluated by 26.5 percent as positive or very positive for investor protection, by 44.2 percent as neutral, and by 29.2 percent of advisors as negative or very negative. Thus, there is no consensus to what extent cost transparency helps investors.

The client-advisor relationship. A growing literature argues that the relationship to the financial advisor is important for the client who seeks investment advice. Gennaioli et al. (2015) argue that the main role of financial advisors is not to provide information, but to act as "money doctors" who help clients making risky investments by reducing their anxiety about taking risk. They "are trusted to do so even when their advice is costly, generic, and occasionally self-serving" (Gennaioli et al. 2015, page 92). Consumers who seek advice are willing to pay substantial fees on investment products if they have sufficient trust in their advisor. Therefore, surprise charges are neither necessary nor productive.

Given that firms benefit substantially from their clients' trust, it is of no surprise that they are interested in maintaining a good relationship with their clients. Hackethal and Inderst (2012) document for a sample of German companies that most firms measure consumers' satisfaction with the service and the probability of recommentation of the service to others. However, they mostly focus on emotional components, and not on judgments about investment performance. They also employ "mystery shopping" to monitor the quality of the service.

Further, Kostovetsky (2016) shows that for retail investors the relationship to the fund management matters. If the ownership of a mutual fund changes, there are significant outflows of investments following the announcement date. This effect is particularly pronounced for mutual funds with high management fees. Hence, for the consumers who invest into these funds the relationship to the asset management is especially important. Further, Choi and Robertson (2020) find that some of the most important factors that determine the share invested in equity is trust in market participants and trust in financial advisors.

6 Conclusion

Consumer behavior in many markets with complex pricing schedules is difficult to reconcile with rationality and standard preferences. To study such markets, it has been assumed that a share of consumers does not take the add-on charges of certain services into account. While such an assumption is reasonable for some one-time transactions, it is less convincing when consumers use a product frequently, or when firms need to build up reputation and thus wish to avoid large discrepancies between consumer expectations and actual experiences.

We therefore studied a market with complex pricing schedules where boundedly rational consumers correctly anticipate their expenses in equilibrium, but falsely predict how these expenses would change if they make different product or effort choices. The core results from the classic papers on hidden add-on pricing obtain in this model, but only if the market is sufficiently competitive. We showed that firms have limited incentives to invest into exploitative innovation, and that continuous exploitative innovation creates incentives to improve products even if innovation is non-appropriable. These innovation incentives are strong enough to improve welfare in the shrouding equilibrium beyond the rational consumer benchmark level. Further, our model implies that transparency regulation is ineffective as long as it does not update the boundedly rational consumers' thinking about outcomes off the equilibrium path. In contrast, limits on maximal add-on prices are effective. Both observations are in line with empirical findings in the market for credit/debit cards and in the market for mutual funds.

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A Appendix

A.1 Boundedly Rational Expectations

We first derive the expectations of a boundedly rational consumer about the distribution over price-states at a given strategy q. Then we characterize their features and how they depend on the consumer's subjective model $\mathcal{R}$.

Derivation of Boundedly Rational Expectations. The derivation of beliefs follows the Bayesian network model as introduced by Spiegler (2016). Suppose the consumer adopts strategy q with $q(a = 2) = \alpha$, $q(a = 1) = \beta$, and $q(a = 0) = 1 - \alpha - \beta$. According to the objective model $\mathcal{R}^*$, the joint distribution over all variables is given by the factorization formula

$$q(a, x_1, x_2, x_3) = q(a)q(x_1 \mid a)q(x_2 \mid a, x_1)q(x_3 \mid x_1, x_2). \tag{14}$$

The objective beliefs about how product choice affects the distribution over price-states equal

$$q(x_3 \mid a) = \sum_{x_1 \in \{0,1\}} \sum_{x_2 \in \{0,1\}} q(x_1 \mid a)q(x_2 \mid a, x_1)q(x_3 \mid x_1, x_2). \tag{15}$$

A boundedly rational consumer fits her model $\mathcal{R}$ to the data generated by strategy q. According to her model, the joint distribution over all variables is given by

$$q_{\mathcal{R}}(a, x_1, x_2, x_3) = q(a)q(x_1 \mid a)q(x_2 \mid x_1)q(x_3 \mid x_1, x_2). \tag{16}$$

We derive the conditional probability $q(x_2 \mid x_1)$ that the consumer observes at a given strategy q. To this end, we first need to know the distribution over product choice a when base good use takes on a certain value x_1. From Bayes' rule, we obtain

$$q(a = 2 \mid x_1) = \frac{\alpha q(x_1 \mid a = 2)}{\sum_{a \in A} q(a)q(x_1 \mid a)}, \tag{17}$$

$$q(a = 1 \mid x_1) = \frac{\beta q(x_1 \mid a = 2)}{\sum_{a \in A} q(a)q(x_1 \mid a)}, \tag{18}$$

$$q(a = 0 \mid x_1) = \frac{(1 - \alpha - \beta)q(x_1 \mid a = 2)}{\sum_{a \in A} q(a)q(x_1 \mid a)}. \tag{19}$$

We now can calculate the association between base good and add-on use at given q,

$$q(x_2 \mid x_1) = \sum_{a \in A} q(a \mid x_1)q(x_2 \mid a, x_1). \tag{20}$$

Note that this conditional probability depends on q. However, in the boundedly rational consumer's mind, $q(x_2 \mid x_1)$ is independent of q. We now can derive the boundedly rational consumer's beliefs

$$q_{\mathcal{R}}(x_3 \mid a; q) = \sum_{x_1 \in \{0,1\}} \sum_{x_2 \in \{0,1\}} q(x_1 \mid a) q(x_2 \mid x_1) q(x_3 \mid x_1, x_2). \tag{21}$$

From this equation we directly obtain the relationships in (4) and (5). The equality in (4) implies that the boundedly rational consumer does not understand that the distribution over price-states depends on product choice. Note that product choice only appears in (21) in the conditional probability $q(x_1 \mid a)$. This association is by assumption the same for both products, $q(x_1 \mid a = 2) = q(x_1 \mid a = 1)$ for any x_1, which implies the equality in (4).

The feature of correct expectations on the equilibrium path in (3) directly follows from the fact that the subjective model $\mathcal{R}$ is "perfect": There is no node in this model so that the links from two other nodes point toward it and these two other nodes are not connect in $\mathcal{R}$. As long as $\mathcal{R}$ has this feature, the equality in (3) holds (see Proposition 2 in Spiegler 2017). The intuition for this result is that according to $\mathcal{R}$ the boundedly rational consumer takes into account the correlation between any two variables in her model that have a joint influence on a third variable. Thus, she does not exhibit "neglect of correlation", which could bias her estimates. Schumacher and Thysen (2021) and Ellis and Thysen (2021) further exploit this property of the Bayesian network framework extensively.

The subjective model $\mathcal{R}$. An attractive feature of the Bayesian network framework is that it allows to study how different models of causal reasoning $\mathcal{R}$ affect the boundedly rational consumer' beliefs. We briefly show that in our setting, different subjective models can lead to the same subjective beliefs, and hence to the same equilibrium outcomes.

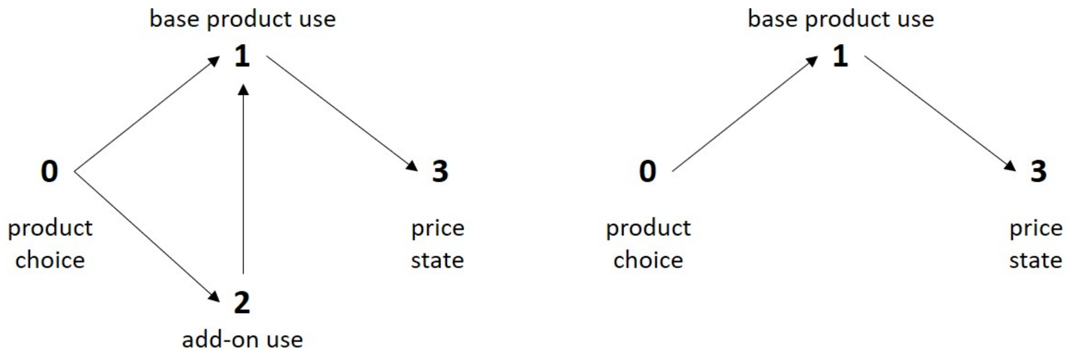

Figure 6: Alternative subjective models $\mathcal{R}_1$ (left) and $\mathcal{R}_2$ (right).

Consider the two alternative subjective models $\mathcal{R}_1$ and $\mathcal{R}_2$ in Figure 6. Under subjective model $\mathcal{R}_1$, the consumer believes that add-on use may have an effect on base good use, but only base good use affects the distribution over price-states. Under subjective model $\mathcal{R}_2$, the consumer ignores add-on use altogether. Using a result from the Bayesian network literature (Proposition 7 from Schumacher and Thysen 2021), we can show that these two models generate the same subjective beliefs than the original subjective model $\mathcal{R}$, that is

$$q_{\mathcal{R}_1}(x_3 \mid a; q) = q_{\mathcal{R}_2}(x_3 \mid a; q) = q_{\mathcal{R}}(x_3 \mid a; q), \tag{22}$$

for any q, a. Thus, our results hold for different types of misspecifications in the consumers' subjective model. Note from the subjective model $\mathcal{R}_2$ that this result implies that the boundedly rational consumer's knowledge about the add-on in $\mathcal{R}$ has no effect on her beliefs.

There are of course subjective models that can generate beliefs which differ from those implied by $\mathcal{R}$. One example would be if the boundedly rational consumers' model is identical to the objective model $\mathcal{R}^*$ except that the link between base good and add-on use is missing. Consumers would then ignore the correlation between base good and add-on use which may lead to biased estimates about the price-state distribution. Formally, this means that the consumer's subjective model is no longer perfect. The property that beliefs are unbiased on the equilibrium path may then no longer hold.

A.2 Omitted Proofs from Subsection 3.1 and Subsection 3.2

Proof of Proposition 1. Since the w-product offers more gains from trade than the v-product, only the w-product is traded in equilibrium when all consumers are rational. Therefore, we ignore prices for the v-product in the following. **Step 1.** We derive the symmetric equilibrium outcome when $\frac{t}{n} < \frac{2}{3}(w - c^w)$. This is the standard case where the consumer who is indifferent between two neighboring firms i and j at prices p_i^w, p_j^w is defined by

$$w - p_i^w - td = w - p_j^w - t\left(\frac{1}{n} - d\right). \tag{23}$$

From this we get that if all other firms charge p_{-i}^w, then demand for firm i's w-product is $D_i = \frac{p_{-i}^w - p_i^w}{t} + \frac{1}{n}$, and that there is a symmetric equilibrium in which all firms charge $p^w = c^w + \frac{t}{n}$. Indeed, the consumer at distance $\frac{1}{2n}$ to the two closest firms then trades at this price if $\frac{t}{n} < \frac{2}{3}(w - c^w)$. **Step 2.** We show that the equilibrium outcome is unique when $\frac{t}{n} < \frac{2}{3}(w - c^w)$. Assume by contradiction that there is an equilibrium in which some firm i charges a price $p_i^w > c^w + \frac{t}{n}$. Assume w.l.o.g. that firm i charges the highest price. We show that firm i can deviate profitably by lowering its price. We consider firm i's profits $\tilde{\pi}_i$ from trading with consumers on the interval between firm i and any neighboring firm j. We distinguish between two cases, Case (A) and Case (B). In Case (A), the marginal consumer on this interval who trades with firm i is indifferent between trading with firm i and not trading at all. This consumer is characterized by the equality

$$w - p_i^w - td = 0 \tag{24}$$

The profit $\tilde{\pi}_i$ decreases in p_i^w if $p_i^w > \frac{w+c^w}{2}$, and it is maximal only if $p_i^w \leq \frac{w+c^w}{2}$. If $p_i^w \leq \frac{w+c^w}{2}$, then, by the assumption that $\frac{t}{n} < \frac{2}{3}(w - c^w)$, it must be the case that the marginal consumer for firm i is located further than $\frac{1}{2n}$ away from this firm, which contradicts the assumption that firm i charges the highest price. Thus, we must have $p_i^w > \frac{w+c^w}{2}$. In Case (B), the marginal consumer on the interval between firm i and firm j is indifferent between the two firms and earns strictly positive surplus. This consumer is characterized by the equality in (23). We then have

$$\tilde{\pi}_i = (p_i^w - c^w)\left(\frac{p_j^w - p_i^w}{2t} + \frac{1}{2n}\right), \tag{25}$$

and $\frac{\partial \tilde{\pi}_i}{\partial p_i^w} < 0$ if

$$c^w + \frac{t}{n} < 2p_i^w - p_j^w, \tag{26}$$

which, by assumption, is satisfied. Combining the results from Case (A) and Case (B) shows that firm i can profitably lower its price, a contradiction. Hence, there cannot be an equilibrium in which any firm i charges a price $p_i^w > c^w + \frac{t}{n}$. Next, assume by contradiction that there is an equilibrium in which some firm i charges a price $p_i^w < c^w + \frac{t}{n}$. Assume w.l.o.g. that firm i charges the lowest price. We show that firm i can deviate profitably by increasing its price. By the arguments above, we must have $p_j^w \leq c^w + \frac{t}{n}$ for all firms j in this equilibrium. We again consider firm i's profits $\tilde{\pi}_i$ from trading with consumers on the interval between firm i and any neighboring firm j. The marginal consumer on this interval must be indifferent between trading with firm i and firm j, and earn strictly positive surplus. This consumer is defined by the equality in (23) and $\tilde{\pi}_i$ is given by equation (25). Hence, we have $\frac{\partial \tilde{\pi}_i}{\partial p_i^w} > 0$ if

$$c^w + \frac{t}{n} > 2p_i^w - p_j^w, \tag{27}$$

which, by assumption, is satisfied. Thus, firm i can profitably increase its price, a contradiction. This completes the proof of the statement. **Step 3.** We derive the symmetric equilibrium outcome when $\frac{2}{3}(w - c^w) \leq \frac{t}{n} \leq w - c^w$. Note that when all firms charge the price $p^w = w - \frac{t}{2n}$, the consumers at distance $\frac{1}{2n}$ to the two closest firms are indifferent between trading or not. We show that if $\frac{2}{3}(w - c^w) \leq \frac{t}{n}$, it does not pay off for firm i to undercut this price by some $\epsilon > 0$ when all other firms also charge it. If firm i makes this change, the marginal consumers for firm i are defined by

$$w - \left(w - \frac{t}{2n} - \epsilon\right) - td = w - \left(w - \frac{t}{2n}\right) - t\left(\frac{1}{n} - d\right), \tag{28}$$

demand for firm i's w-product is $D_i = \frac{1}{n} + \frac{\epsilon}{t}$, and firm i's profit equals

$$\pi_i = \left(w - \frac{t}{2n} - \epsilon - c^w\right)\left(\frac{1}{n} + \frac{\epsilon}{t}\right). \tag{29}$$

Note that $\frac{2}{3}(w - c^w) \leq \frac{t}{n}$ implies $\frac{\partial \pi_i}{\partial \epsilon} < 0$ at any $\epsilon > 0$. Similarly, we can show that if $\frac{t}{n} \leq w - c^w$, it does not pay off for firm i to charge a price higher than $p^w = w - \frac{t}{2n}$. Hence, if $\frac{2}{3}(w - c^w) \leq \frac{t}{n} \leq w - c^w$, then there is an equilibrium where each firm charges $p^w = w - \frac{t}{2n}$ and serves $\frac{1}{n}$th of the market. Standard arguments show that there exists no other symmetric equilibrium outcome. **Step 4.** We derive the symmetric equilibrium outcome when $w - c^w < \frac{t}{n}$. Consider any firm i and suppose that it is a monopolist. If it charges the price p_i^w, the distance d to the consumer who is indifferent between trading or not is defined by $w - p_i^w - td = 0$. Demand for firm i's w-product is then $D_i = \frac{2(w - p_i^w)}{t}$, from which we obtain the optimal price $p^w = \frac{w + c^w}{2}$. At this price, the marginal consumers for firm i are located less than $\frac{1}{2n}$ away from it if and only if $w - c^w < \frac{t}{n}$. By construction, if this inequality holds, it is optimal for all firms to charge $p^w = \frac{w + c^w}{2}$. Standard arguments show that this is also the unique equilibrium outcome. This completes the proof. □

Proof of Proposition 2, statements (i), (ii), (iv) and (v). We prove the four statements in steps. **Step 1.** We prove statement (i). Suppose that $\frac{t}{n} < p_{add}^v - c^v$ and consider an assessment where firms shroud add-on prices, all rational consumers purchase the w-product at price $p^w = c^w + \frac{t}{n}$, and all boundedly rational consumers purchase the v-product at price $p^v = p_{add}^v$ (which means that the base price of the v-product is zero). Since $c^w > p_{add}^w$, this implies that boundedly rational consumers strictly prefer the v-product from firm i to the w-product from firm i. The assumption on the maximal add-on price ensures that no consumer earns a negative payoff from following the proposed purchase plan. Note that, in this assessment, firms then earn strictly more from selling a v-product than from selling a w-product. Thus, no firm can gain by educating consumers. Standard arguments show that firms also cannot profit from charging different prices. Hence, the considered assessment is an equilibrium. **Step 2.** We prove statement (ii). Suppose that $p_{add}^v - c^v \leq \frac{t}{n} \leq \frac{2}{3}(v - c^v)$ and $\tilde{p} > (w - c^w) - (v - c^v)$. Consider an assessment where firms shroud add-on prices, all rational consumers purchase the w-product at price $p^w = c^w + \frac{t}{n}$, and all boundedly rational consumers purchase the v-product at price $p^v = c^v + \frac{t}{n}$. The assumption on $\tilde{p}$ ensures that boundedly rational consumers strictly prefer the v-product from firm i to the w-product from firm i. The assumption on $\frac{t}{n}$ ensures that no consumer earns a negative payoff from following the proposed purchase plan. Firms earn the same profit from trading the w-product and the v-product. Prices are determined by competition between firms as in the symmetric benchmark equilibrium from Proposition 1. Thus, no firm can gain by educating consumers or by charging different prices. Hence, the considered assessment is an equilibrium. **Step 3.** We prove statement (iv). Suppose that $\frac{2}{3}(w - c^w) < \frac{t}{n} \leq w - c^w$. Assume by contradiction that there is a symmetric equilibrium in which firms trade the v-product. As in the proof of statement (iii) of Proposition 2 below, we can show that in this equilibrium firms sell the w-product to rational consumers and the v-product to boundedly rational consumers. Note from Proposition 1 that the symmetric equilibrium profits from trading the v-product are strictly below the symmetric equilibrium profits from trading the w-product. Hence, the equilibrium price p^w for the w-product must weakly exceed the symmetric benchmark equilibrium price $w - \frac{t}{2n}$. If it were lower, each firm would have an incentive to increase this price (note that it could do so without disturbing trade with boundedly rational consumers). Consequently, each firm could profitably deviate by educating consumers and selling the w-product to all consumers at the symmetric benchmark equilibrium price, a contradiction. The last part of statement (iv) follows from standard arguments. Finally, the proof of statement (v) is straightforward and therefore omitted. □

Proof of Proposition 2, statement (iii). We prove the statement in several steps. **Step 1**. We show that in an equilibrium in which the v-product is traded, each firm also sells the w-product to a positive share of rational consumers. Observe that the v-product can only be traded in equilibrium when all firms shroud their add-on prices (otherwise, all consumers would be rational and only the w-product would survive competitive pressure and profit maximization). Assume by contradiction that there is an equilibrium in which firm i only sells the v-product at total price p_i^v to consumers. In equilibrium, we must have that firm i serves a positive fraction of rational as well as boundedly rational consumers, and earns a positive profit so that $p_i^v > c^v$. Suppose firm i now offers the w-product at total price

$$p_i^w = p_i^v - (v - w) - \epsilon. \tag{30}$$

If ϵ is small enough, the following holds: At least those rational consumers who trade with firm i now purchase the w-product instead of the v-product. Since $w - c^w > v - c^v$, this increases firm i's total profit, a contradiction. **Step 2**. We make two important observations. First, note that at any given position on the circle, it cannot happen in equilibrium that a boundedly rational consumer purchases a w-product, while a rational consumer at the same position purchases a v-product. The reason is that the boundedly rational consumer would then strictly underestimate the total price of the v-product and switch to this product. Second, observe that, by the tie-breaking rule, in any equilibrium in which the v-product is traded, all boundedly rational consumers who trade with firm i purchase the w-product if $w-p^w_{i,1} \geq v-p^v_{i,1}$, and purchase the v-product if $w - p^w_{i,1} < v - p^v_{i,1}$. **Step 3**. We show that, in an equilibrium in which the v-product is traded, there is no firm i that charges a price p^w_i below the rational benchmark level $c^w + \frac{t}{n}$. Assume by contradiction that there exists an equilibrium in which firm i charges $p^w_i < c^w + \frac{t}{n}$; w.l.o.g. we assume that firm i offers the w-product at the lowest price, $p^w_i \leq p^w_j$ for any other firm j. Let firms j, k be firm i's neighbors. By Step 1 and Step 2, firm i's profit then equals

$$\pi_i = \lambda(p^u_i - c^u)(D^{br}_{ij} + D^{br}_{ik}) + (1-\lambda)(p^w_i - c^w)(D^r_{ij} + D^r_{ik}), \tag{31}$$

where $u \in \{w, v\}$ denotes the product that boundedly rational consumers trade with firm i (according to Step 2), and D^r_{ij} (D^{br}_{ij}) denotes the mass of rational (boundedly rational) consumers who trade with firm i and who are located on the interval between firm i and firm j; D^r_{ik}, D^{br}_{ik} denote the same for consumers located on the interval between firm i and firm k. In Step 4 to Step 7, we show that either firm i can increase π_i by charging a higher price for the w-product or one of firm i's neighbors can increase its profit by charging a different price for the w-product. **Step 4**. Suppose there is a boundedly rational consumer who in equilibrium (i) purchases the w-product from firm i, (ii) is indifferent between the w-product from firm i and the v-product from firm j or firm k, and (iii) strictly prefers trading to not trading. By Step 2, statement (i) implies that firm i then only sells the w-product to consumers. Note that the consumer overestimates the utility from purchasing the v-product. Hence, by continuity and statements (ii) and (iii), firm i could increase its profit by advertising add-on prices. The described situation therefore cannot occur in equilibrium. **Step 5**. Suppose the marginal consumers who purchase the w-product from firm i (i) are indifferent between the w-product from firm i and the w-product from firm j or firm k, and (ii) strictly prefer trading to not trading. We show that in this case firm i can increase its profit by raising p^w_i by some small $\epsilon > 0$. The marginal consumer on the segment between firm i and firm j is defined by the equality

$$w - p^w_i - \epsilon - td = w - p^w_j - t\left(\frac{1}{n} - d\right), \tag{32}$$

and hence by

$$d = \frac{p^w_j - p^w_i - \epsilon}{2t} + \frac{1}{2n}. \tag{33}$$

The profit that firm i is making from selling the w-product to consumers located between firm

i and firm j (normalized by the share of rational/boundedly rational consumers) equals

$$\tilde{\pi}_i = (p_i^w + \epsilon - c^w)\left(\frac{p_j^w - p_i^w - \epsilon}{2t} + \frac{1}{2n}\right). \quad (34)$$

Observe that $\tilde{\pi}_i$ strictly increases in ϵ if

$$-p_j^w + 2(p_i^w + \epsilon) < c^w + \frac{t}{n}. \quad (35)$$

By construction, we have $p_i^w \leq p_j^w$, which implies that this inequality is satisfied if ϵ is sufficiently small. Finally, note that increasing p_i^w by a small amount would not affect firm i's sales of its v-product, which completes the proof of the statement. **Step 6**. Suppose that, in equilibrium, there exists a firm j that charges a price $p_j^w > c^w + \frac{t}{n}$ so that the marginal consumers who purchase the w-product from firm j are indifferent between trading or not trading. We show that it cannot be the case that the boundedly rational consumers who trade with firm j (i) purchase the v-product, (ii) are arbitrary close to indifference between purchasing the w-product and the v-product from firm j, and (iii) firm j earns weakly more out of boundedly rational consumers than out of rational consumers. Assume by contradiction that this is the case. Since boundedly rational consumers who purchase the v-product overestimate the price of the w-product, statement (ii) implies that the marginal boundedly rational consumer who purchases the v-product is closer to firm j than the marginal rational consumer who purchases the w-product. The fact that $\frac{t}{n} < \frac{2}{3}(w - c^w)$ and $w - c^w > v - c^v$ then imply that statement (iii) cannot be true. **Step 7**. Suppose the marginal consumer on the segment between firm i and firm j who purchases the w-product from firm i is indifferent between this product and not trading at all. This implies that the marginal consumer on the same segment who purchases the w-product from firm j is also indifferent between this product and not trading at all. Moreover, since firm i charges the lowest price for the w-product, this also implies that the marginal consumer on the segment between firm j and its other neighbor, firm j', who purchases the w-product from firm j is also indifferent between this product and not trading at all. Note that firm j's marginal consumers for the w product are located less than $\frac{1}{2n}$ away from firm j. We show that firm j then can increase its profit by lowering its price p_j^w by some small ϵ. Since $\frac{t}{n} < \frac{2}{3}(w - c^w)$, this is true if after the price adjustment the marginal consumers are still indifferent between trading or not trading the w-product with firm j. Assume therefore w.l.o.g. that after the price adjustment the marginal consumer on the segment between firm j and firm j' is defined by

$$w - p_j^w + \epsilon - td = w - p_{j'}^w - t\left(\frac{1}{n} - d\right). \quad (36)$$

The profit that firm j is making from selling the w-product to consumers located between firm

j and firm j' (normalized by the share of rational/boundedly rational consumers) equals

$$\tilde{\pi}_j = (p_j^w - \epsilon - c^w)\left(\frac{p_{j'}^w - p_j^w + \epsilon}{2t} + \frac{1}{2n}\right). \tag{37}$$

We can show that $\tilde{\pi}_j$ strictly increases in ϵ if

$$2(p_j^w - \epsilon) - p_{j'}^w > c^w + \frac{t}{n}. \tag{38}$$

The location of marginal consumers implies that $p_{j'}^w < p_j^w$. Hence, the inequality in (38) is satisfied if ϵ is sufficiently small. The result then follows from Step 6 when we apply the same steps to the segment between firm i and firm j and take into account that firm j can also choose to educate boundedly rational consumers. Taken together, Step 5 and Step 7 imply that, in an equilibrium in which the v-product is traded, there is no firm i that charges a price $p_i^w < c^w + \frac{t}{n}$.
Step 8. We show that an equilibrium in which the v-product is traded cannot exist. Consider first any firm i that only sells the w-product to consumers (if such a firm exists). Note that the share of boundedly rational consumers it serves must be at least as large as the share of rational consumers, $D_{ij}^{br} + D_{ik}^{br} \geq D_{ij}^{r} + D_{ik}^{r}$, otherwise firm i could gain by educating consumers. Moreover, since all of firm i's rivals charge at least $c^w + \frac{t}{n}$ for the w-product, the optimal price for firm i is such that the fraction rational consumers it serves is at least $\frac{1}{n}$. Next, consider any firm j that sells the v-product to boundedly rational consumers. Since all firms charge at least $c^w + \frac{t}{n}$ for the w-product, this firm can earn at least the symmetric benchmark equilibrium profit out of rational consumers. Since $\frac{t}{n} > \frac{2}{3}(v - c^v)$, it earns weakly more out of boundedly rational consumers only if the fraction of boundedly rational consumers it serves exceeds $\frac{1}{n}$. By the observations above, this cannot be the case for all firms that sell the v-product. Hence, at least one of them could profitably educate consumers and charge different prices. This completes the proof. □

Proof of Proposition 3. The statement that firm 1 does not invest into innovation if all consumers are rational directly follows from the firm profits in the benchmark equilibrium as described in Proposition 1. Statement (i) follows from the fact that in the symmetric shrouding equilibrium and in any symmetric equilibrium with advertised add-on prices the markup on the w-product is $\frac{t}{n}$, so that the firms' profits from the w-product are independent of $w - c^w$. We prove statement (ii). Suppose that firm i conducts the v-innovation project. If $\tilde{v} - \tilde{c}^v > w - c^w$, then, given the prices in the symmetric shrouding equilibrium, any firm i may have an incentive to advertise add-on prices and to sell only the (now superior) v-product. We derive the optimal total price for this deviation, assuming that all other firms $-i$ charge the symmetric shrouding equilibrium prices. When all consumers are rational, they strictly prefer the w-product from a firm $j \neq i$ to the v-product from firm j. We derive the maximal price p_i^v firm i can charge for the v-product so that it serves (almost) all consumers. If n is odd, this price equals $p_i^v = \tilde{v} - (w - c^w) - \frac{t}{n}\frac{n+1}{2}$, and the corresponding profit (if positive) equals

$$\pi_i = (\tilde{v} - \tilde{c}^v) - (w - c^w) - \frac{t}{n}\frac{n+1}{2}. \tag{39}$$

If n is even, this price equals $p_i^v = \tilde{v} - (w - c^w) - \frac{t}{n}\frac{n+2}{2}$, and the corresponding profit (if positive) equals

$$\pi_i = (\tilde{v} - \tilde{c}^v) - (w - c^w) - \frac{t}{n}\frac{n+2}{2}. \tag{40}$$

The optimal total price may be larger than these prices. Define by $x \geq 1$ the number of segments between firm i and the firm furthest away from firm i so that firm i still serves consumers in segment x. If n is odd, the maximal value of x equals $\frac{n-1}{2}$, and if n is even, the maximal value of x equals $\frac{n}{2}$. If firm i charges a price p_i^v so that it does not serve all consumers, then the marginal consumers is characterized by

$$\tilde{v} - p_i^v - td = w - \left(c^w + \frac{t}{n}\right) - t\left(\frac{x}{n} - d\right), \tag{41}$$

and firm i's profit equals

$$\pi_i = (p_i^v - \tilde{c}^v)\left(\frac{(\tilde{v} - p_i^v) - (w - c^w)}{t} + \frac{1}{n}(1 + x)\right). \tag{42}$$

Thus, if the price p_i^v is optimal, it satisfies

$$p_i^v = \frac{\tilde{v} - w + \tilde{c}^v + c^w}{2} + \frac{t}{2n}(1 + x), \tag{43}$$

and the corresponding profit equals

$$\pi_i = \frac{1}{t}\left[\frac{(\tilde{v} - \tilde{c}^v) - (w - c^w)}{2} + \frac{t}{2n}(1 + x)\right]^2, \tag{44}$$

where x is the largest value so that, at the beginning of segment x, consumers prefer the v-product of firm i to the w-product of their closest firm, i.e.,

$$x = \left\lfloor [(\tilde{v} - \tilde{c}^v) - (w - c^w)]\frac{n}{t} + 1 \right\rfloor. \tag{45}$$

The equations (39), (40), and (44) define the maximal profit $\pi^{un}(\tilde{v}, \tilde{c}^v)$ from the proposed deviation. We can verify that $\pi^{un}(\tilde{v}, \tilde{c}^v) = 0$ if $\tilde{v} - \tilde{c}^v = w - c^w$, $\pi^{un}(\tilde{v}, \tilde{c}^v)$ strictly increases in $\tilde{v} - \tilde{c}^v$, and that, as $\tilde{v} - \tilde{c}^v$ increases, there is at most one intersection with $\tilde{\pi}^{sh} = \frac{1}{n}\left[\lambda(\tilde{v} - \frac{t}{2n} - \tilde{c}^v) + (1 - \lambda)\frac{t}{n}\right]$, the profit from compliance in the symmetric shrouding equilibrium. As soon as the proposed deviation becomes profitable, the symmetric shrouding equilibrium no longer exists, and products are traded with markup $\frac{t}{n}$ in any symmetric continuation equilibrium. From this statement (ii) follows. □

A.3 The Model without Education

We consider the model from Section 2, but assume that firms cannot advertise add-on prices to educate consumers. Firms only set prices and the behavior of boundedly rational consumers must be a personal equilibrium according to Definition 1. We analyze the set of equilibria in this new game and obtain the following result.

Proposition 5 (Equilibrium in the Model without Education)**.** *Suppose firms cannot advertise add-on prices. Assume that there is a share of boundedly rational consumers, $\sqrt{\frac{1}{2}}(v - c^v) \leq \frac{2}{3}(w - c^w)$, and that the maximal add-on price is small enough such that $v - p^v_{add} \geq \frac{1}{3}(v - c^v)$.*

(i) If $\frac{t}{n} < v - p^v_{add}$ and $\tilde{p} > (w-c^w)-(v-c^v)$, then (a) there exists no equilibrium in which only the w-product is traded, and (b) in the unique symmetric equilibrium, rational consumers purchase the w-product at price $p^w = c^w + \frac{t}{n}$ and boundedly rational consumers purchase the v-product at price $p^v = p^v_{add}$.

(ii) If $v - p^v_{add} \leq \frac{t}{n} < \frac{2}{3}(v-c^v)$ and $\tilde{p} > (w-c^w)-(v-c^v)$, then (a) there exists no equilibrium in which only the w-product is traded, and (b) in the unique symmetric equilibrium, rational consumers purchase the w-product at price $p^w = c^w + \frac{t}{n}$ and boundedly rational consumers purchase the v-product at price $p^v = c^v + \frac{t}{n}$.

(iii) If $\frac{2}{3}(v - c^v) \leq \frac{t}{n} < \sqrt{\frac{1}{2}}(v - c^v)$ and $\tilde{p} > (w - c^w) - \frac{5}{6}(v - c^v)$, then (a) there exists no equilibrium in which only the w-product is traded, and (b) in the unique symmetric equilibrium, rational consumers purchase the w-product at price $p^w = c^w + \frac{t}{n}$ and boundedly rational consumers purchase the v-product at price $p^v = v - \frac{t}{2n}$.

(iv) If $\sqrt{\frac{1}{2}}(v-c^v) \leq \frac{t}{n}$, there exists a symmetric equilibrium in which all consumers purchase the w-product at the prices indicated in Proposition 1. This is also the equilibrium that maximizes industry profits.

The proof of Proposition 5 is presented below. The two qualifications at the beginning of the proposition are again not essential, they only save us from more case distinctions. Proposition 5 contains a number of important observations. First, firms may sell only the superior product to consumers even though add-on prices remain shrouded. Specifically, this can happen when firms have sufficient market power, see statement (iv). Starting from a situation where all firms sell the w-product at symmetric equilibrium prices, a single firm then cannot profit from selling the inferior product. Intuitively, its appeal to boundedly rational consumers does not outweigh the reduced gains from trade. Hence, market power holds back firms from introducing inferior products.

When the market is sufficiently competitive and the inferior product appears sufficiently attractive to boundedly rational consumers relative to the superior product (i.e., $\tilde{p}$ is large enough), such an equilibrium does not exist, see statements (i) to (iii). In a situation where only the w-product is sold, each firm would have an incentive to introduce the v-product to exploit its advantage with boundedly rational consumers.

Second, in a symmetric equilibrium, firms benefit from the existence of the v-product only if the market is sufficiently competitive, i.e., when $\frac{t}{n} < v - p^v_{add}$, see statement (i). Only then are symmetric equilibrium profits strictly larger than in the benchmark equilibrium. For intermediate degrees of competition, firms either earn the same profit as in the symmetric benchmark equilibrium, see statement (ii), or strictly less, see statement (iii). Hence, firms have incentives to develop and introduce the v-product only if competition is fierce enough.

Overall, the differences between the model with costless education and the model without education are modest. With costless education, there always exists an equilibrium in which only the w-product is traded, while without education this is only the case when there is sufficiently little competition. It seems plausible that firms coordinate on an equilibrium where they benefit from shrouding equilibrium prices if such benefits exist in equilibrium. Further, with costless education, the v-product is no longer traded when there is sufficiently little competition between firms. In contrast, without education it still can be optimal for firms to sell the inferior product to boundedly rational consumers, even when they enjoy monopoly power. The reason is that boundedly rational consumers have pessimistic beliefs in a personal equilibrium when firms only offer the w-product. These consumers then overestimate the probability of the high price-state after purchasing the w-product and therefore may prefer not trading at all to purchasing the w-product. Not learning the true probability of the high price-state of the w-product then leaves these beliefs unchallenged.

Proof of Proposition 5. We prove statements (ii) to (iv) in steps. The proof of statement (i) uses very similar arguments as the proof of statement (ii) in Steps 1 and 2 and is therefore omitted. **Step 1**. We show that if $v - p^v_{add} \leq \frac{t}{n} < \frac{2}{3}(v - c^v)$ and $\tilde{p} > (w - c^w) - (v - c^v)$, then there exists no equilibrium in which only the w-product is traded. By Proposition 1, each firm would charge $p^w = c^w + \frac{t}{n}$ and earn total profits of $\pi = \frac{t}{n^2}$ in such an equilibrium. We show that a firm then can deviate profitably by selling the v-product to some boundedly rational consumers.

Suppose firm i sells the v-product at base price $p^v_{i,base}$. All boundedly rational consumers at a distance $d \leq \frac{1}{2n}$ to firm i weakly prefer the v-product from firm i to the w-product from firm i if

$$v - p^v_{i,base} - p^v_{add} \geq w - \left(c^w + \frac{t}{n} - p^w_{add}\right) - p^v_{add}, \tag{46}$$

which can be rewritten as

$$p^v_{i,base} \leq v + \frac{t}{n} - (w - c^w) - p^w_{add}. \tag{47}$$

These consumers also weakly prefer to purchase the v-product from firm i to not purchasing anything if

$$v - p^v_{i,base} - p^v_{add} - \frac{t}{2n} \geq 0, \tag{48}$$

which can be re-arranged as

$$p^v_{i,base} \leq v - \frac{t}{2n} - p^w_{add}. \tag{49}$$

Suppose firm i sells the v-product at the base price defined by the right-hand side of inequality (47). The profit from boundedly rational consumers is then larger than under the original situation where only the w-product is sold if

$$\left((v - c^v) - (w - c^w) + \frac{t}{n} + p^v_{add} - p^w_{add}\right)\frac{1}{n} > \frac{t}{n^2}, \tag{50}$$

which is equivalent to the assumption that $\tilde{p} > (w - c^w) - (v - c^v)$. Next, assume that the base price is defined by the right-hand side of inequality (49). Firm i's profit from boundedly rational consumers is then larger than under the original situation where only the w-product is sold if

$$\left(v - c^v - \frac{t}{2n}\right)\frac{1}{n} > \frac{t}{n^2}, \tag{51}$$

which is equivalent to the assumption that $\frac{t}{n} < \frac{2}{3}(v - c^v)$. The two observations taken together imply that there exists a profitable deviation for firm i, which completes the proof of the statement. **Step 2.** We show that if $v - p^v_{add} \leq \frac{t}{n} < \frac{2}{3}(v - c^v)$ and $\tilde{p} > (w - c^w) - (v - c^v)$, then there exists a symmetric equilibrium in which rational consumers purchase the w-product at price $p^w = c^w + \frac{t}{n}$ and boundedly rational consumers purchase the v-product at price $p^v = c^v + \frac{t}{n}$. Note that the assumption on $\tilde{p}$ ensures that boundedly rational consumers then indeed purchase the v-product. By Proposition 1, the only potentially profitable deviation for firms is to make the v-product so expensive so that boundedly rational consumers prefer the w-product from firm i to the v-product from firm i. However, there is no price p^w that firm i could charge to make this change profitable (note that such boundedly rational consumers still would need to weakly prefer the w-product from firm i to the w-product and v-product from any other firm). **Step 3.** We show that if $\frac{2}{3}(v - c^v) \leq \frac{t}{n} < \sqrt{\frac{1}{2}}(v - c^v)$ and $\tilde{p} > (w - c^w) - \frac{5}{6}(v - c^v)$, then there exists no equilibrium in which only the w-product is traded. By Proposition 1 and the assumption that $\sqrt{\frac{1}{2}}(v - c^v) \leq \frac{2}{3}(w - c^w)$, each firm would charge $p^w = c^w + \frac{t}{n}$ and earn total profits of $\pi = \frac{t}{n^2}$ in such an equilibrium. We show that firm i can deviate profitably by selling the v-product to boundedly rational consumers. Suppose firm i charges the base price $p^v_{i,base}$ for the v-product, sells it to all boundedly rational consumers up to distance d, and that the marginal boundedly rational consumer is indifferent between purchasing the v-product from firm i and not trading at all. We then have

$$v - p^v_{i,base} - p^v_{add} - td = 0, \tag{52}$$

so that $p^v_{i,base} = v - p^v_{add} - td$. This deviation is profitable if

$$2(v - c^v - td)d > \frac{t}{n^2}. \tag{53}$$

From this inequality we get the critical distance

$$d^* = \frac{(v - c^v) + \frac{1}{2}\sqrt{(v - c^v)^2 - 2\left(\frac{t}{n}\right)^2}}{2t}. \tag{54}$$

There is a profitable deviation where firm i sells the v-product to boundedly rational consumers if, at $p^v_{i,base} = v - p^v_{add} - td^*$, the boundedly rational consumers at distance d^* (i) strictly prefer the v-product from firm i to the w-product from the neighboring firm and (ii) strictly prefer the v-product from firm i to the w-product from firm i. Condition (i) holds if

$$v - p^v_{i,base} - p^v_{add} - td^* > w - \left(c^w + \frac{t}{n} - p^v_{add}\right) - p^v_{add} - t\left(\frac{1}{n} - d^*\right). \tag{55}$$

This inequality is equivalent to

$$\tilde{p} > (w - c^w) + \frac{1}{2}(v - c^v) - \frac{2t}{n}. \tag{56}$$

Since $\frac{t}{n} \geq \frac{2}{3}(v - c^v)$, this inequality is implied by the assumption on $\tilde{p}$. Condition (ii) holds if

$$v - p^v_{i,base} > w - \left(c^w + \frac{t}{n} - p^v_{add}\right). \tag{57}$$

Again, this inequality follows from $\frac{t}{n} \geq \frac{2}{3}(v - c^v)$ and the assumption on $\tilde{p}$. This completes the proof of the statement. **Step 4.** We show that if $\frac{2}{3}(v - c^v) \leq \frac{t}{n} < \sqrt{\frac{1}{2}}(v - c^v)$ and $\tilde{p} > (w - c^w) - \frac{5}{6}(v - c^v)$, then there exists a symmetric equilibrium in which rational consumers purchase the w-product at price $p^w = c^w + \frac{t}{n}$ and boundedly rational consumers purchase the v-product at price $p^v = v - \frac{t}{2n}$. The assumption on $\tilde{p}$ ensures that, at these prices, boundedly rational consumers indeed purchase the v-product. The only potentially profitable deviation of any firm i is to increase the base price of the v-product such that boundedly rational consumers prefer the w-product of firm i to the v-product of firm i. We show that no such deviation is profitable by deriving an upper bound on the profit that firm i can make from selling the w-product to boundedly rational consumers. To get this upper bound, we assume that firm i can price discriminate between rational and boundedly rational consumers. We distinguish between two cases, Case (i) and Case (ii). Case (i): Assume that under the optimal base price $p^w_{i,base}$ charged to boundedly rational consumers, the marginal boundedly rational consumer is indifferent between purchasing the w-product from firm i and not trading at all. The boundedly rational consumers' beliefs imply that the marginal consumers are then characterized by the equality

$$w - p^w_{i,base} - p^v_{add} - td = 0. \tag{58}$$

The profit from selling the w-product to boundedly rational consumers is then

$$\pi_i = 2(p^w_{i,base} + p^w_{add} - c^w)\left(\frac{w - p^w_{i,base} - p^v_{add}}{t}\right). \tag{59}$$

The price that maximizes this profit is $p^w_{i,base} = \frac{w+c^w}{2} - \frac{p^v_{add}+p^w_{add}}{2}$, and the corresponding profit equals

$$\pi_i = \frac{2}{t}\left(\frac{w - c^w - \tilde{p}}{2}\right)^2. \tag{60}$$

We compare this profit to the profit firm i would make from boundedly rational consumers on the equilibrium path. The deviation is not profitable if

$$\left(\frac{w - c^w - \tilde{p}}{2}\right)^2 \leq \frac{1}{2}\frac{t}{n}(v - c^v) - \frac{1}{4}\left(\frac{t}{n}\right)^2. \tag{61}$$

From the assumption on the marginal boundedly rational consumer we get

$$\frac{w - p^w_{i,base} - p^v_{add}}{t} \leq \frac{1}{2n}, \tag{62}$$

which implies that $\tilde{p} \geq w - c^w - \frac{t}{n}$. We use this inequality to re-write (61) and get that it is satisfied if $\frac{t}{n} \leq v - c^v$. By assumption, we have $\frac{t}{n} \leq \sqrt{\frac{1}{2}}(v - c^v)$, which proves the statement for Case (i). Case (ii): Assume that under the optimal base price $p^w_{i,base}$ charged to boundedly rational consumers, the marginal boundedly rational consumer is indifferent between purchasing the w-product from firm i and purchasing the v-product from a neighboring firm. The marginal consumer is then characterized by the equality

$$w - p^w_{i,base} - td = v - \left(c^v + \frac{t}{n} - p^v_{add}\right) - t\left(\frac{1}{n} - d\right). \tag{63}$$

So the marginal consumer is located at

$$d = \frac{(w - p^w_{i,base}) - (v - c^v) - p^v_{add}}{2t} + \frac{1}{n}. \tag{64}$$

The profit from selling the w-product to boundedly rational consumers is then

$$\pi_i = 2(p^w_{i,base} + p^w_{add} - c^w)\left(\frac{(w - p^w_{i,base}) - (v - c^v) - p^v_{add}}{2t} + \frac{1}{n}\right). \tag{65}$$

The price that maximizes this profit is $p^w_{i,base} = \frac{t}{n} + \frac{w+c^w}{2} - \frac{v-c^v}{2} - p^w_{add} - p^v_{add}$, and the corresponding profit equals

$$\pi_i = \frac{1}{t}\left(\frac{t}{n} + \frac{(w - c^w) - (v - c^v) - \tilde{p}}{2}\right)^2. \tag{66}$$

We compare this profit to the profit firm i would make from boundedly rational consumers on the equilibrium path. The deviation is not profitable if

$$\left(\frac{t}{n} + \frac{(w - c^w) - (v - c^v) - \tilde{p}}{2}\right)^2 \leq \frac{t}{n}(v - c^v) - \frac{1}{2}\left(\frac{t}{n}\right)^2. \tag{67}$$

The assumption on $\tilde{p}$ and $\frac{t}{n} < \sqrt{\frac{1}{2}}(v - c^v)$ ensure that this inequality is satisfied, which proves the statement for Case (ii). **Step 5.** We show that if $\sqrt{\frac{1}{2}}(v - c^v) \leq \frac{t}{n}$, then there exists a symmetric equilibrium in which all consumers purchase the w-product at the prices indicated in Proposition 1. Assume first that $\frac{t}{n} \leq \frac{2}{3}(w - c^w)$. Consider an assessment where all firms charge $p^w = c^w + \frac{t}{n}$ for the w-product, a base price for the v-product above v (so that boundedly rational consumers do not purchase it), and all consumer purchase the w-product. We show that no firm can profitably deviate from this assessment by changing the price of the v-product. Suppose firm i reduces the price of the v-product so that boundedly rational consumers purchase it. In the best case, the marginal boundedly rational consumers are indifferent between purchasing the v-product from firm i and not trading at all. These consumers would be characterized by

$$v - p^v_{i,base} - p^v_{add} - td = 0. \tag{68}$$

The optimal base price would then be $p^v_{i,base} = \frac{v+c^v}{2} - p^v_{add}$, and the corresponding profit from boundedly rational consumers (normalized by their share)

$$\pi_i = \frac{(v - c^v)^2}{2t}. \tag{69}$$

We compare this profit to the profit firm i makes from boundedly rational consumers under the original assessment. By Proposition 1, this value is $\frac{t}{n^2}$. Since $\sqrt{\frac{1}{2}}(v - c^v) \leq \frac{t}{n}$, there is no profitable deviation, which proves the claim for $\frac{t}{n} \leq \frac{2}{3}(w - c^w)$. Similarly, we can show that there exists a symmetric equilibrium in which all consumers purchase the w-product at the prices indicated in Proposition 1 when $w - c^w > \frac{t}{n} \geq \frac{2}{3}(w - c^w)$ and when $\frac{t}{n} > w - c^w$. In the former case, this follows from the assumption that $\sqrt{\frac{1}{2}}(v - c^v) \leq \frac{2}{3}(w - c^w)$. Finally, the last statement follows from the fact that firms sell the w-product at the equilibrium price that maximizes industry profits. □

A.4 Omitted Proofs from Section 4

Proof of Lemma 1. We prove statement (i). Assume that the marginal rational and boundedly rational consumers are indifferent between the two neighboring firms and weakly prefer trading to not trading. If all other firms charge the base price $p_{-i,base}$, then demand for firm i's v-product from both rational and boundedly rational consumers at base price $p_{i,base}$ is $D_i = \frac{p_{-i,base} - p_{i,base}}{t} + \frac{1}{n}$, and firm i's profit equals

$$\pi_i = \lambda(p_{i,base} + p_{add} - c)\left(\frac{p_{-i,base} - p_{i,base}}{t} + \frac{1}{n}\right) + (1 - \lambda)(p_{i,base} - c)\left(\frac{p_{-i,base} - p_{i,base}}{t} + \frac{1}{n}\right). \tag{70}$$

From this we get that, in a symmetric equilibrium, each firm charges the base price $p_{base} = c - \lambda p_{add} + \frac{t}{n}$. We check until what value of transport costs all consumers indeed purchase the product at this price. Since $p_{add} > e$, this is the case if

$$v - p_{base} - p_{add} - \frac{t}{2n} \geq 0, \tag{71}$$

which is equivalent to

$$\frac{2}{3}[(v - c) - (1 - \lambda)p_{add}] \geq \frac{t}{n}. \tag{72}$$

This proves the result. The uniqueness of the symmetric equilibrium follows from standard arguments which show that at a higher (lower) base price p_{base} firms have an incentive to cut (increase) the base price. This also holds for the other statements and will not be repeated. We prove statement (ii). Suppose that $\frac{t}{n}$ is large enough so that it violates the threshold in (72). We show that if $\frac{t}{n}$ is not too large, then, in the symmetric equilibrium, firms charge a base price so that they serve all consumers, and the marginal boundedly rational consumers are indifferent between trading and not trading. By definition, this base price must be equal to $p_{base} = v - p_{add} - \frac{t}{2n}$. If firm i deviates and increases the price by some small $\epsilon > 0$, its profit equals

$$\pi_i = 2\lambda(p_{base} + p_{add} + \epsilon - c)\frac{v - p_{base} - p_{add} - \epsilon}{t} + (1-\lambda)(p_{base} + \epsilon - c)\left(\frac{1}{n} - \frac{\epsilon}{t}\right). \tag{73}$$

Differentiating this expression with respect to ϵ gets us that firms have no incentive to charge a higher price if

$$\frac{2}{3+\lambda}[(1+\lambda)(v-c) - (1-\lambda)p_{add}] \geq \frac{t}{n}. \tag{74}$$

Standard arguments show that firms also cannot profitably charge a lower price, which completes the proof of statement (ii). We prove statement (iii). Suppose that $\frac{t}{n}$ is large enough so that it violates the threshold in (74). Assume that the marginal rational consumers are indifferent between the two neighboring firms and strictly prefer trading to not trading, while the marginal boundedly rational consumers are indifferent between trading with the closest firm and not trading at all. We derive the symmetric equilibrium price for this setting. Firm i's profit from charging $p_{i,base}$ when all other firms charge $p_{-i,base}$ equals

$$\pi_i = 2\lambda(p_{i,base} + p_{add} - c)\frac{v - p_{i,base} - p_{add}}{t} + (1-\lambda)(p_{i,base} - c)\left(\frac{p_{-i,base} - p_{i,base}}{t} + \frac{1}{n}\right). \tag{75}$$

From the corresponding first-order condition we get that, in the unique symmetric equilibrium, firms charge

$$p_{base} = \frac{2\lambda(v + c - 2p_{add}) + (1-\lambda)(c + \frac{t}{n})}{1 + 3\lambda}. \tag{76}$$

We can check that under this price the marginal boundedly rational consumers are indeed indifferent between trading with the closest firm and not trading at all, given that (74) is violated. The assumption on p_{add} ensures that some boundedly rational consumers still purchase the good under the base price in (76). Finally, we check until what value of transport costs all rational consumers purchase the product at this price. This is the case if

$$v - p_{base} - e - \frac{t}{2n} \geq 0, \tag{77}$$

which is equivalent to

$$\frac{2}{3+\lambda}[(1+\lambda)(v-c) + 4\lambda p_{add} - (1+3\lambda)e] \geq \frac{t}{n}. \tag{78}$$

This completes the proof of statement (iii). We prove statement (iv). Suppose that $\frac{t}{n}$ is large enough so that it violates the threshold in (78). We show that if $\frac{t}{n}$ is not too large, then in the symmetric equilibrium firms charge a base price so that they serve all rational consumers, and the marginal rational and boundedly rational consumers are indifferent between trading or not trading. By definition, this base price must be equal to $p_{base} = v - e - \frac{t}{2n}$. The assumption on p_{add} ensures that some boundedly rational consumers still purchase the good under this price. If firm i deviates and increases the price by some small $\epsilon > 0$, its profit equals

$$\pi_i = 2\lambda(p_{base} + p_{add} + \epsilon - c)\frac{v - p_{base} - p_{add} - \epsilon}{t} + 2(1-\lambda)(p_{base} + \epsilon - c)\frac{v - p_{base} - e - \epsilon}{t}. \quad (79)$$

Differentiating this expression with respect to ϵ gets us that firms have no incentive to charge a higher price if

$$(v - c) + 2\lambda p_{add} - (1+\lambda)e \geq \frac{t}{n}. \quad (80)$$

Standard arguments show that firms also cannot profitably charge a higher price, which completes the proof of statement (vi). We prove statement (v). Suppose $\frac{t}{n}$ is large enough so that it violates the threshold in (80). Assume that the marginal rational and boundedly rational consumers are indifferent between trading with the closest firm and not trading at all. We derive the equilibrium price for this setting. Firm i's profit from charging $p_{i,base}$ equals

$$\pi_i = 2\lambda(p_{i,base} + p_{add} - c)\frac{v - p_{base} - p_{add}}{t} + 2(1-\lambda)(p_{i,base} - c)\frac{v - p_{i,base} - e}{t}. \quad (81)$$

From the corresponding first-order condition we get that, in the unique symmetric equilibrium, firms charge

$$p_{base} = \frac{v+c}{2} - \frac{2\lambda p_{add} + (1-\lambda)e}{2}. \quad (82)$$

We can check that under this price the marginal rational and boundedly rational consumers are indeed indifferent between trading with the closest firm and not trading at all, given that (80) is violated. The assumption on p_{add} ensures that some boundedly rational consumers still purchase the good under the price in (76), which completes the proof of statement (v). □

Proof of Proposition 4. We prove statement (i). Assume that it is indeed optimal for firms to shroud add-on prices. By Lemma 1 and the assumption $\frac{2}{3}[(v-c) - p_{add}] \geq \frac{t}{n}$, we must have that, in the symmetric shrouding equilibrium, firms charge the base price $p_{base} = c - \lambda p_{add} + \frac{t}{n}$, the maximal add-on price p_{add}, and each firm earns a profit of

$$\pi^{sh} = \frac{t}{n^2}. \quad (83)$$

Suppose firm i unshrouds its add-on price and charges w.l.o.g. an add-on price of zero. We derive the optimal base-price. If firm i charges the base price $p_{i,base}$ and this base price is not too large, the marginal consumers for firm i are defined by

$$v - p_{i,base} - td = v - p_{base} - t\left(\frac{1}{n} - d\right). \tag{84}$$

The share of consumers it serves then equals $D_i = \frac{c - \lambda p_{add} + \frac{t}{n} + e - p_{i,base}}{t} + \frac{1}{n}$. Its profit from charging $p_{i,base}$ is $\pi_i = (p_{i,base} - c)D_i$, and the optimal base price equals

$$\tilde{p}_{i,base} = c + \frac{t}{n} - \frac{\lambda p_{add} - e}{2}. \tag{85}$$

Hence, the highest possible profit from deviation is

$$\pi^{un} = \left(\frac{t}{n} - \frac{\lambda p_{add} - e}{2}\right)\left(\frac{1}{n} - \frac{\lambda p_{add} - e}{2t}\right). \tag{86}$$

From equations (83) and (86) we get that no firm can deviate profitably from the suggested symmetric shrouding equilibrium if $\lambda p_{add} \geq e$, which completes the proof of the statement. We prove statement (ii). We prove it separately for three cases: Case (i) $\frac{t}{n} < \frac{2}{3}(v - c)$, Case (ii) $\frac{2}{3}(v - c) \leq \frac{t}{n} < v - c$, and Case (iii) $v - c \leq \frac{t}{n} < (v - c) + p_{add} - e$. Consider Case (i). Suppose that $\lambda \approx 1$ and that it is indeed optimal for firms to shroud add-on prices. Lemma 1 then shows that, in the symmetric shrouding equilibrium, firms charge the base price $p_{base} \approx c - p_{add} + \frac{t}{n}$, the maximal add-on price p_{add}, and each firm serves the share $\frac{1}{n}$ of rational and boundedly rational consumers. As in the proof of statement (i) we compute the optimal prices of firm i after unshrouding the add-on price. At $\lambda \approx 1$ the optimal base price equals

$$\tilde{p}_{i,base} \approx c - \frac{p_{add} - e}{2} + \frac{t}{n} \tag{87}$$

given that the add-on price is set to zero. This value is strictly smaller than $p_{base} + p_{add}$. The marginal consumers are then at distance

$$d \approx \frac{1}{2n} - \frac{\tilde{p}_{i,base} - (c - p_{add} + \frac{t}{n})}{2t} < \frac{1}{2n} \tag{88}$$

to their closest firm. Hence, by continuity, if λ is sufficiently large, a firm cannot profitably unshroud its add-on price. Next, consider Case (iii). Suppose that $\lambda \approx 1$ and that it is indeed optimal for firms to shroud add-on prices. Lemma 1 then shows that, in the symmetric shrouding equilibrium firms, charge the base price $p_{base} \approx \frac{v+c}{2} - p_{add}$, the maximal add-on price p_{add}, and hence earn approximately monopoly profits. Marginal boundedly rational consumers are located at distance

$$d \approx \frac{v - c}{2t}. \tag{89}$$

If firm i unshrouds its add-on price and charges the same total price as before then for $\lambda \approx 1$ the marginal boundedly rational consumer (and those boundedly rational consumers close to her) trade with firm i's rivals if

$$v - p_{base} - e - t\left(\frac{1}{n} - \frac{v-c}{2t}\right) > 0, \tag{90}$$

which for $\lambda = 1$ is equivalent to

$$\frac{t}{n} < (v-c) + p_{add} - e. \tag{91}$$

Hence, if λ is close enough to 1, then a firm cannot profitably unshroud its add-on price as its profit would be bounded away from monopoly profits. Finally, the proof for Case (ii) is very similar to that for Case (iii) and therefore omitted. We prove statement (iii). Assume by contradiction that a shrouding equilibrium exists. In this equilibrium, firms charge the maximal add-on price as well as the base price $p_{base} = \frac{v+c}{2} - \frac{2\lambda p_{add}+(1-\lambda)e}{2}$ if it is optimal for them to serve both rational and boundedly rational consumers, and $p_{base} = \frac{v+c}{2} - \frac{e}{2}$ if it is optimal for them to serve only rational consumers. Moreover, rational consumers exert substitution effort in this equilibrium. Assume for a moment that firm i is the only firm in the market and all consumers are boundedly rational. It would then earn maximal profits by unshrouding its add-on price and charging $p_{i,base} = \frac{v+c}{2}$ and $p_{i,add} = 0$. The sum of these prices would also be the unique optimal total price. Next, assume that firm i is the only firm in the market and all consumers are rational. Again, firm i would then earn the maximal profit by unshrouding its add-on price and charging the same prices (and their sum would again be the unique optimal total price). Taking together these observations implies that firm i can profitably deviate by unshrouding its add-on price and charging $p_{i,base} = \frac{v+c}{2}$ and $p_{i,add} = 0$, a contradiction. □