Equilibrium Contracts and Boundedly Rational Expectations: Supplementary Appendix

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A A Brief Introduction to d-separation

We briefly introduce the concept of d-separation, a result from the Bayesian network literature that allows us to check, for any given model $\mathcal{R}$, whether two variables (or two sets of variables) are independent when conditioning on a third variable (or set of variables). For simple models $\mathcal{R}$ it can be used as visual inspection tool; for complex models, there exists an algorithm for checking d-separation (Geiger et al. 1990). Define a path τ in $\mathcal{R}$ as a sequence of nodes so that any adjacent nodes are linked in $\mathcal{R}$; τ is a directed path if the links between any two adjacent nodes in τ point in the same direction (from the former to the latter or vice versa). A node j is a descendant of node i if there exists a directed path from i to j. For convenience, we use the notation $i \to j$ instead of iRj in this section. The following definitions and result are adopted from Pearl (2009).

Definition 8. *A path τ is blocked in $\mathcal{R} = (R, N)$ by a set of variables $M \subset N$ if and only if one of the following condition holds:*

(a) τ contains variables i, m, j with $m \in M$ so that $i \to m \to j$ or $i \leftarrow m \to j$, or

(b) τ contains variables i, m, j so that $i \to m \leftarrow j$, $m \notin M$, and no descendant of m is in M.

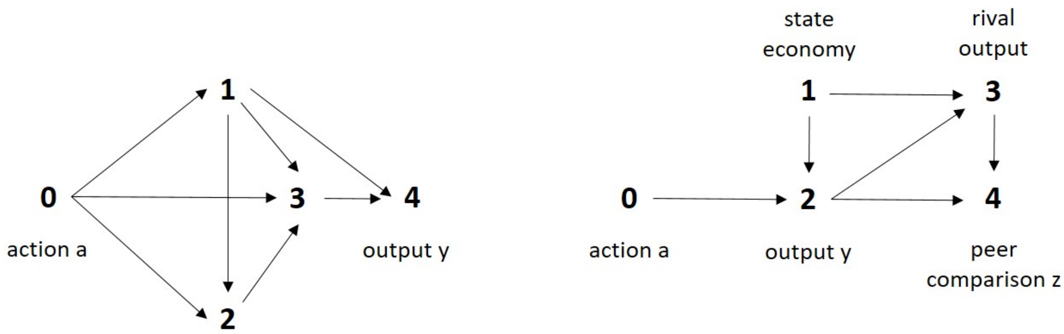

Figure A1: Objective model $\mathcal{R}^*$ from Figure 1 (left) and objective model $\mathcal{R}^*$ from Figure 3 (right).

To illustrate, consider the DAG $\mathcal{R}^*$ from Figure 1 in the paper, reproduced here on the left of Figure A1. The path $\tau = 0 \to 2 \leftarrow 1 \to 3 \to 4$ between the nodes 0 and 4 is blocked by node 1 and node 3, but not by node 2. To see this, note that conditions (a) and (b) are both satisfied if we define $M = \{1\}$, or $M = \{3\}$; however, none of the conditions is satisfied if we define $M = \{2\}$.

Definition 9. *Let $\mathcal{R} = (R, N)$ be a DAG and M', M'', M disjoint subsets of N. M' and M'' are d-separated by M in $\mathcal{R}$, if M blocks every path between any node in M' and any node in M''.*

Consider the DAG $\mathcal{R}^*$ from Figure 3 in the paper, reproduced here on the right of Figure A1. We check whether the nodes 0 and 4 are d-separated in $\mathcal{R}^*$ by $M = \{2\}$. For this, we have to consider three paths, $\tau = 0 \to 2 \to 4$, $\tau' = 0 \to 2 \leftarrow 1 \to 3 \to 4$, and $\tau'' = 0 \to 2 \to 3 \to 4$. By condition (a) in Definition 8, the paths τ and τ'' are blocked by $M = \{2\}$. In contrast, the path τ' is not blocked by $M = \{2\}$. Hence, the nodes 0 and 4 are not d-separated in $\mathcal{R}^*$ by $M = \{2\}$. However, they are d-separated in $\mathcal{R}^*$ by $M = \{1, 2\}$, $M = \{2, 3\}$, or $M = \{1, 2, 3\}$. Suppose, for example, that $M = \{1, 2\}$. Now not only the paths τ and τ'' are blocked according to condition (a) in Definition 1, but also path τ' (we see this from the segment $2 \leftarrow 1 \to 3$). The implication of d-separation is given in the following result.

Proposition 10 (Implications of d-separation)**.** *If the variables* 0 *and n are d-separated by variable i in $\mathcal{R}$, then $p_{\mathcal{R}}(x_n \mid x_0, x_i; q) = p_{\mathcal{R}}(x_n \mid x_i; q)$ for all $q \in \Delta(A)$ and all triples x_0, x_i, x_n. If the variables* 0 *and n are not d-separated by variable i in $\mathcal{R}$, then x_0 and x_n are dependent conditional on x_i for at least one distribution compatible with $\mathcal{R}$.*

References

GEIGER, DAN, THOMAS VERMA, AND JUDEA PEARL (1990), "Identifying independence in Bayesian networks." *Networks*, 20, 507–534.

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