The Agency Costs of On-the-Job Search*

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Version: February 16, 2019

Abstract

This paper studies how workers' on-the-job search influences optimal incentives in organizations. We analyze a principal-agent model in which the agent multitasks between working for the principal and searching for other job opportunities. The agent partly uses on-the-job search to improve his bargaining position within the relationship. We show that the optimal contract may feature both excessive performance bonuses as well as efficiency wages. Both measures reduce the agent's search incentives, but do not completely eliminate rent-seeking under the optimal contract. On-the-job search therefore generates agency costs. The model suggests a new rational for excessive incentive pay and efficiency wages.

Keywords: Repeated Games, On-the-Job Search, Multitasking, Efficiency Wages
JEL Classification: C73, D83, M52

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*We are grateful to Ricardo Alonso, Dan Barron, Mark Bernard, Oscar Contreras, Matthias Fahn, Daniel Ferreira, Guido Friebel, Susanne Goldlücke, Sergei Guriev, Florian Hett, Heiko Karle, Michael Kosfeld, Matthias Kräkel, Albert Ma, Jim Malcomson, Francesca Melillo, Mike Powell, Michael Raith, Luis Rayo, Klaus Schmidt, Dirk Sliwka, Patrick van Cayseele and seminar participants at Carlos III de Madrid, University of New South Wales, Ludwig-Maximilians University, CUNEF, HU Berlin, University of Utah, Vanderbilt University, Pompeu Fabra, London School of Economics, Goethe-University Frankfurt, University of Cologne, University of Trier, Gothenburg (EEA meetings), Düsseldorf (VfS meetings), Aarhus (SMYE), Tübingen (COPE) and the 2015 Workshop on Relational Contracts in Munich for valuable comments and discussions. The usual disclaimer applies.

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1 Introduction

On-the-job search is a widespread practice in the labor market. Workers in all kinds of occupations and positions frequently look for alternative employment opportunities. Since the mid 1980s, many labor markets have experienced a steep increase in job-to-job transition rates (Stewart 2002). Fallick and Fleischman (2004) show that there are about twice as many workers who change employers than workers who move from unemployment to employment; on-the-job search explains employment-to-employment transitions just as well as off-the-job search explains transitions from non-employment to employment.[1] Job-to-job transitions seem to be especially common in high-technology clusters such as the computer industry in Silicon Valley (Fallick et al. 2006) as well as among consulting and professional service companies.

However, searching for a new job can be very time-consuming, involving activities such as attending job training courses, screening job advertisements, writing applications, preparing for job interviews, as well as traveling and meeting human resource representatives or employment agencies. Mueller (2010) reports that employed job seekers devote more than 100 minutes to search activities during each day of their search. Arellano and Meghir (1992) find that on-the-job search has a negative impact on hours worked. Moreover, to appear attractive to other employers, job seekers may shift their attention away from productive, less visible tasks, towards tasks that are less productive but more easily observed by outsiders. Therefore, employers may be concerned that their employees' on-the-job search impairs organizational performance.

In this paper, we examine how on-the-job search affects the provision of incentives inside organizations. We consider a repeated principal-agent model in which the agent (he) faces a multitasking problem (Holmström and Milgrom 1991). He can work for the principal (she) or search for outside options. Performing one activity drives up the costs of the other activity. The principal-agent relationship creates economic gains through the agent's work on the project and his opportunity to work on the project until he finds a more attractive outside option. This framework provides novel perspectives on a variety of contractual arrangements as well as on recent changes in the incentive contracts which are employed in practice.

Our first key observation is that on-the-job search has mixed blessings. A certain level of on-the-job search is socially valuable as it leads to a better worker-firm match with positive probability. The first-best level of on-the-job search is therefore positive. However, finding a more valuable match is not the agent's only benefit from search effort. By generating outside

[1] See also Pissarides and Wadsworth (1994), Stevenson (2008), and Ahn and Shao (2017) for empirical evidence on the importance of on-the-job search for labor market dynamics. Highly skilled employees may also search for opportunities to start-up their own business; see the literature on the transition from employment to self-employment, e.g., Elfenbein et al. (2010) or Carnahan et al. (2012).

offers, the agent improves his bargaining position within the current relationship. Securing outside offers is valuable for the agent – even when they represent less productive matches – since they help him to negotiate better terms with the principal.[2] Yet, on-the-job search for such offers is socially wasteful, as it creates effort costs, but only leads to a redistribution of the joint surplus.

How does the agent's on-the-job search affect the optimal incentive contract? In the absence of on-the-job search, the principal would realize all gains from trade by paying the agent the full marginal returns of his efforts and extracting all rents through a low fixed-wage. This contract is no longer optimal once the agent engages in on-the-job search. Relative to the first-best, the agent would invest too much into search because of his rent-seeking motive. We show that the optimal incentive contract exhibits two features that reduce excessive search incentives. The first feature is an excessive bonus for high output that reduces search through the well-known static effort-substitution effect. Since the principal can keep the agent's expected payoff constant by adjusting the wage, increasing the bonus above full marginal returns results in first-order gains, but only second-order costs. Nevertheless, very large bonus payments are costly for the principal since she has to compensate the agent for the costs of excessive effort. Hence, the effectiveness of excessive bonuses to reduce search is limited.

The second feature of the optimal incentive contract is more subtle and stems from the dynamic nature of on-the-job search. If the principal always extracts all rents from the agent, the agent has large incentives to invest in search since even low-value offers improve his bargaining position. The principal can further curb search incentives by promising the agent a continuation payoff above the value of very unattractive outside options. This promise can be credible when the expected duration of the relationship is sufficiently long. Essentially, the principal then pays an efficiency wage to reduce rent-seeking through on-the-job search. Thus, the optimal incentive contract may exhibit both excessive bonus payments and efficiency wages. Both features reduce search incentives. However, since the principal cannot credibly promise the agent the entire value of the relationship, search incentives remain inefficiently high under the optimal contract. On-the-job search therefore creates agency costs.

Our paper offers a new perspective on the increase in the use of performance pay in many occupations. Since the 1980s, both the share of workers who receive some type of performance pay (piece rates, bonuses, or stock options), and the importance of performance pay relative to fixed wages, has increased significantly; see, for example, Lazear and Shaw (2007). This development contributed substantially to the rise in wage inequality. There are several different explanations for the increase in the use of performance pay. First, the increase in the

[2] Employers frequently match offers from competing firms to retain their best workers; see Barron et al. (2006) and Yamaguchi (2010).

returns to skill (due to technological changes) may induce firms to offer performance pay in order to attract more talented workers. Lemieux et al. (2009) show that workers' total compensation is more closely related to their productive characteristics if they receive performance pay, which is consistent with the sorting hypothesis. Alternatively, steep increases in the use of performance may not reflect technological changes, but are more likely a result of poor corporate governance; see Bertrand and Mullainathan (2001). The prevalence of this view is also illustrated by the increase in compensation-related regulations in recent years.[3]

In this paper, we suggest an extension of the sorting hypothesis. Performance pay may be used to not only attract talented workers, but also to retain them. Many industries experience intense competition for talent in the labor market (Friebel and Giannetti 2009). Workers who face stronger competition for their services can generate better outside offers and thus may enjoy higher returns from on-the-job search. In order to distract workers from wasteful on-the-job search, firms optimally offer excessive performance pay to them.

Our model also offers a new rational for paying efficiency wages. Efficiency wage models describe mechanisms that imply a positive relationship between a worker's wage and his productivity. The most commonly used efficiency wage model in contract theory is that of Shapiro and Stiglitz (1984). In their framework, employers motivate their staff by paying wages above the market-clearing level and firing workers who are caught shirking. The equilibrium wage leads to involuntary unemployment so that losing a job is costly for workers.[4] In our model, the rent that the agent receives does not reward him for good performance, but reduces his wasteful on-the-job search activities and thereby increases productivity. For this reason, the optimal incentive contract may combine efficiency wages with performance pay. This is in contrast to classic models (such as MacLeod and Malcomson 1998) in which firms offer to a worker either efficiency wages or bonus payments.

The remainder of the paper is organized as follows. In Section 2, we relate our contribution to the previous literature. Section 3 describes the model. In Section 4, we examine how on-the-job search changes the optimal incentive contract. In Section 5, we discuss several extensions of the baseline model. Section 6 concludes. All proofs are relegated to the Appendix.

[3]For example, in the European Union, the Capital Requirements Directive (CRD) imposes a bonus cap on the compensation of asset managers. Moreover, several European countries have passed laws capping the variable compensation of managers, while in the US, the Dodd-Frank Act also contains a set of compensation-related regulations.

[4]There exist alternative mechanisms that generate a positive wage/productivity relationship. A high wage may be interpreted as a gift from the employer that the worker reciprocates by increasing his effort (Akerlof 1982). Alternatively, employers may offer high wages to lower turnover (Stiglitz 1974 and Salop 1979) or to increase the quality of their pool of applicants (e.g., Weiss 1980).

2 Related Literature

Our paper contributes to various strands of the contract theory and labor economics literature.

Contract Design with On-the-Job Search. There are two papers that study optimal contract design with moral hazard and on-the-job search.[5] Moen and Rosen (2013) study a two-period principal-agent model in which the agent invests in search to find a better match in the second period. The optimal contract in this setting distorts search incentives below the efficient level through deferred compensation. Board and Meyer-ter Vehn (2015) examine the labor market equilibrium when firms provide effort incentives through the threat of layoffs only (as in Shapiro and Stiglitz 1984) and workers are randomly matched to job offers. They show that this model generates dispersion in wages and productivity even if firms and workers are ex-ante identical. The key difference between our model and these papers is that, in ours, the agent faces a multitasking problem. The optimal incentive contract therefore exhibits properties that do not arise in these models.[6]

A number of papers analyze optimal contracts when agents choose their search effort endogenously and work effort within the organization is not subject to a moral hazard problem; see Mortensen (1978), Postel-Vinay and Robin (2004), and Lentz (2014). They show that rent-seeking on-the-job search can be reduced by back-loading the agent's compensation in long-term contracts. This solution is not available in our framework since the principal cannot commit to long-term contracts.

Relational Contracts. When performance measures are not verifiable to third parties or the agent's performance can only be assessed subjectively, parties may agree on relational contracts, i.e., contracts which rely on repeated games incentives; see Bull (1987), MacLeod and Malcomson (1989), and Levin (2003). Within this literature, the model by Baker et al. (2002) is related to ours in the sense that one party can invest in the alternative use value of its product, which would improve the bargaining position within the relationship. The authors examine how asset ownership changes the relational contract in this setting. A number of papers find non-trivial employment dynamics in stationary environments as we have in our model; see Chassang (2010), Halac (2012), and Li and Matouschek (2013). Fuchs (2007) considers

[5] There is a large literature that examines the impact of on-the-job search on aggregate economic outcomes. Pissarides (1994) studies search unemployment with on-the-job search; Burdett and Mortensen (1998) and Christensen et al. (2005) show that on-the-job search can generate wage and productivity dispersion among ex-ante homogeneous workers; Lentz (2010) analyzes aggregate sorting patterns in the labor market under on-the-job search and Lise (2013) studies how on-the-job search and precautionary savings shape earnings inequality.

[6] Another related paper is Englmaier et al. (2014) who study optimal contracts for "knowledge workers" in a static setting. The agent's effort not only increases the expected output, but also the value of the agent's outside option. As in our setting, the agent may leave the principal too early under the optimal contract. Hvide and Kristiansen (2012) study optimal incentive contracts when knowledge workers may steal ideas from firms that they generated through their work. In both models, there is no multitasking. Hence, the shape of the optimal contract in these models is quite different from that in ours.

a repeated principal-agent model in which only the principal observes the agent's output. The optimal contract in this setting is an efficiency wage contract that motivates the agent through the threat of dismissal. Fuchs (2007) therefore provides a micro-foundation for the efficiency wage model in Shapiro and Stiglitz (1984). Maestri (2012) complements the analysis in Fuchs (2007) by showing that an efficiency wage contract is inferior to a performance pay contract if the agent observes a signal that is sufficiently correlated with his output.

In an extension of our model (Subsection 5.1), we show that our main result also holds in a relational contracts framework, i.e., when the bonus is discretionary. Thus, we contribute to the relational contracts literature by showing that the optimal relational contract may feature both excessive bonuses and efficiency wages when the agent engages in on-the-job search. In the baseline model, we assume that the bonus in a given period is enforceable.

Multitasking. After the seminal paper by Holmström and Milgrom (1991), a large body of literature analyzed optimal incentive contracts when agents perform multiple tasks. The two most closely related papers in this literature are Schöttner (2008) and Benabou and Tirole (2016). Schöttner (2008) examines the optimal combination of explicit and relational contracts in a multitasking setup with infinitely many periods. The different tasks all contribute to the value of the principal's project, while the agent's outside option value is fixed. The optimal contract therefore features neither excessive bonuses nor efficiency wages. The model in Benabou and Tirole (2016) generates excessive bonuses in a static multitasking environment. In their model, firms compete for talent and screen out highly skilled workers by offering performance incentives that exceed the second-best level. In our model, by contrast, the principal offers excessive bonuses to reduce the agent's search incentives. In addition, she may also pay efficiency wages to further reduce the agent's wasteful search activities.

3 The Model

Basic Framework. A principal interacts with an agent in periods $t = 0, 1, ...$ Both parties are risk-neutral and discount future payoffs by the discount factor $\delta \in (0, 1)$. The relationship between principal and agent endures as long as the agent accepts the principal's contract offers. If he accepted all offers until period t, the principal offers a contract (w_t, b_t), where w_t is a fixed wage and b_t is an output-based bonus. Both components are enforceable. We assume that the principal's human capital is needed for the project so that she cannot sell her complete business to the agent in the first period.[7]

[7]For example, she may possess valuable but non-transferable client relationships. Alternatively, the principal may be doing some other necessary work or having some expertise which is not explicitly modeled, but is essential for the productiveness of the relationship.

Denote by $d_t \in \{0, 1\}$ the agent's participation decision in period t. If he rejects the contract ($d_t = 0$), the agent gets the outside option value r_t in period t and the same value in all subsequent periods; in all periods $\tau \geq t$, the principal then earns zero and we have $d_\tau = 0$. If he accepts the contract ($d_t = 1$), he exerts work effort $e_t \in [0, \overline{e}]$ and search effort $s_t \in [0, \overline{s}]$ at cost $c(e_t, s_t)$. The cost function is convex in both arguments, $c_e(e_t, s_t) > 0$ for $e_t > 0$, $c_s(e_t, s_t) > 0$ for $s_t > 0$, $c_{ee}(\cdot) > 0$, $c_{ss}(\cdot) > 0$;[8] marginal effort costs at no effort are zero, $c_e(0, s_t) = c_s(e_t, 0) = 0$; and $\max\{c_{ee}(\cdot), c_{ss}(\cdot)\} > c_{se}(\cdot) = c_{es}(\cdot) > 0$.

The agent's work and search effort choices are unobservable to the principal. Work effort generates output $y_t \in \{\underline{y}, \overline{y}\}$. When the agent exerts effort e_t, the probability of high output ($y_t = \overline{y}$) is given by $f(e_t)$, where f is weakly concave, $f'(\cdot) > 0$, $f''(\cdot) \leq 0$, $f(0) = 0$, and $f(\overline{e}) \leq 1$. The output is publicly observable and verifiable to third parties.[9] Search effort s_t determines the probability $p(s_t)$ with which the agent finds an outside option in the next period. If he does not find an outside option in period t, his outside option value in this period is $r_t = 0$. If he finds an outside option, his outside option value r_t is drawn randomly according to the distribution function G. We assume that G has full support on $(0, \infty)$, a continuous density function g, and finite mean $\mathbb{E}_G[r_t] < \infty$. The function p is linear[10] and has $p(\overline{s}) \leq 1$. In the first period, we assume that $r_0 = 0$.

The period-t payoffs after accepting contract (w_t, b_t) are as follows. If the agent chooses effort (e_t, s_t), then with probability $f(e_t)$ the output is high. In this case, the agent's period payoff is $u_t = w_t + b_t - c(e_t, s_t)$, while the principal's payoff is $v_t = \overline{y} - w_t - b_t$; with the reverse probability, the output is low and the agent's period payoff is $u_t = w_t - c(e_t, s_t)$, while the principal's payoff is $v_t = \underline{y} - w_t$. The agent is not protected by limited liability, so $w_t + b_t$ can be negative. The outside option value r_t is not contractible. However, in each period t, the principal observes r_t and can tailor her contract to it. There is free disposal. The agent can reduce the value of his outside option to any value in $[0, r_t]$. Thus, in equilibrium, the agent's payoff cannot decrease in his outside option value. Figure 1 shows the sequence of events in a period t when the agent has accepted the principal's contract in all previous periods.

Strategies and Equilibrium. Denote the set of publicly observable outcomes in period t by $\varphi_t = (r_t, w_t, b_t, y_t, d_t)$. The history of play up to period t is denoted by $h_t^0 = (\varphi_0, \varphi_1, ..., \varphi_{t-1}, r_t)$. Let H_t^0 be the set of such histories up to period t and $H^0 = \bigcup_{t \geq 0} H_t^0$ the set of all finite histories. The principal's strategy σ_P maps the histories in H^0 into contract offers,

$$\sigma_P : H^0 \to \mathbb{R} \times \mathbb{R}. \tag{1}$$

[8]We use c_e and c_s to denote the partial derivative with respect to the first and second variable, respectively.

[9]In Subsection 5.1, we drop the assumption of verifiable output.

[10]This assumption is not essential for our results, but it simplifies their exposition substantially. Note we can have curvature in the effectiveness of search effort through the cost function.

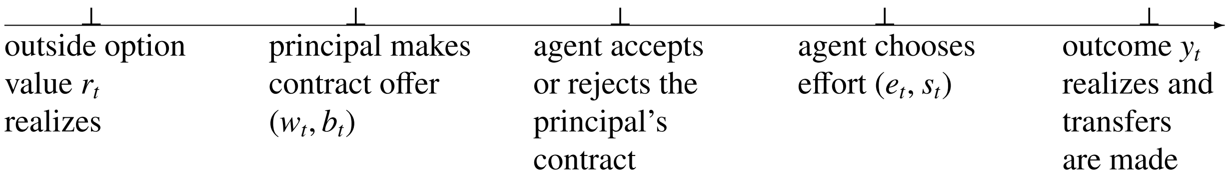

Figure 1: Sequence of events in period t

The agent chooses his action upon observing the contract offer. Denote the history of play up to the point when the agent chooses his action in period t by $h_t^1 = (\varphi_0, \varphi_1, ..., \varphi_{t-1}, r_t, w_t, b_t)$. Let H_t^1 be the set of such histories up to period t and $H^1 = \bigcup_{t \geq 0} H_t^1$. The agent's strategy σ_A maps the histories in H^1 into a participation decision and effort choices,

$$\sigma_A : H^1 \to \{0, 1\} \times [0, \overline{e}] \times [0, \overline{s}]. \tag{2}$$

Let the strategies σ_P and σ_A be given. In period t, the principal's normalized continuation value after history h_t^0 then equals

$$V_t(h_t^0, \sigma_P, \sigma_A) = (1 - \delta)\mathbb{E}\left[\sum_{\tau=0}^{\infty} \delta^\tau d_{t+\tau}(y_{t+\tau} - W_{t+\tau}) \mid h_t^0, \sigma_P, \sigma_A\right], \tag{3}$$

where W_τ is the principal's payment to the agent in period τ. The agent's normalized continuation value after history h_t^1 is given by

$$U_t(h_t^1, \sigma_P, \sigma_A) = (1 - \delta)\mathbb{E}\left[\sum_{\tau=0}^{\infty} \delta^\tau [d_{t+\tau}(W_{t+\tau} - c(e_t, s_t)) + (1 - d_{t+\tau})r_{\tilde{t}}] \mid h_t^1, \sigma_P, \sigma_A\right], \tag{4}$$

where $\tilde{t}$ is the period in which (according to h_t^1) the agent rejected the principal's contract offer. A perfect public equilibrium is a strategy profile $\sigma = (\sigma_P, \sigma_A)$ such that (i) for any $h_t^0 \in H^0$ and any alternative principal strategy $\hat{\sigma}_P$ we have $V_t(h_t^0, \sigma_P, \sigma_A) \geq V_t(h_t^0, \hat{\sigma}_P, \sigma_A)$, and (ii) for any $h_t^1 \in H^1$ and any alternative agent strategy $\hat{\sigma}_A$ we have $U_t(h_t^1, \sigma_P, \sigma_A) \geq U_t(h_t^1, \sigma_P, \hat{\sigma}_A)$. Since the principal makes the contract offers, we are interested in the equilibrium that maximizes her payoff. We will therefore call an equilibrium $\sigma^* = (\sigma_P^*, \sigma_A^*)$ that maximizes $V_0(h_0^0, \sigma_P^*, \sigma_A^*)$ an "optimal incentive contract."[11]

[11] Throughout the paper, when we describe features of the optimal incentive contract, we refer to features that it has "almost always" (i.e., for convenience, we do not use the term "almost always").

4 The Optimal Incentive Contract

4.1 Preliminaries

Our first step is to simplify the analysis by restricting attention to equilibria in which the output y_t affects the principal's and the agent's payoff only in period t, but not in periods $\tau > t$. This separation result follows from risk-neutrality. Any variation in expected future payments that implements a certain level of work effort e_t can be generated through the proper choice of the bonus b_t. This simplifies the analysis substantially.

Lemma 1. *Consider an equilibrium σ in which the principal implements effort $e_t > 0$ after some history h_t^0 through a variation in payments after period t. Then there exists an equilibrium $\hat{\sigma}$ that is payoff equivalent and in which the principal implements e_t after history h_t^0 by conditioning only the agent's payoff in period t on y_t.*

With this result we can focus, without loss of generality, on equilibria in which the agent's total expected utility U_t only depends on his outside option value r_t and not on previous outcomes. Denote by $U_t(r_t)$ the agent's total expected payoff in period t in such an equilibrium as a function of his outside option value r_t. Let $V_t(r_t)$ be corresponding payoff for the principal. Given that the agent works for the principal in period t, he chooses his work and search effort to maximize

$$(1-\delta)(w_t + f(e_t)b_t - c(e_t, s_t)) + \delta(1 - p(s_t))U_{t+1}(0) + \delta p(s_t)\mathbb{E}_G[U_{t+1}(r_{t+1})]. \tag{5}$$

So he influences his continuation value only through his search effort. Differentiating this expression with respect to work and search effort yields two first-order conditions that characterize the agent's equilibrium actions:[12]

$$f'(e_t)b_t = c_e(e_t, s_t), \tag{6}$$

$$\frac{\delta}{1-\delta}p'(s_t)(\mathbb{E}_G[U_{t+1}(r_{t+1})] - U_{t+1}(0)) = c_s(e_t, s_t). \tag{7}$$

Denote by $Q_t(r_t) = V_t(r_t)+U_t(r_t)$ the joint surplus in period t if the agent works for the principal in this period and his outside option value is r_t. The joint surplus consists of two parts: the surplus from working in the principal's project and the (expected) surplus from accepting the outside option in the future. Suppose that in all periods t the agent chooses work effort e_t, search effort s_t, and the participation decision d_t so as to maximize the joint surplus. Note that

[12]Throughout, we assume that the marginal costs of work and search effort rise quickly enough so that the agent's choices are given by interior solutions.

the agent's decision problem is the same in each period in which he accepts the principal's offer. Therefore, there exists a unique value Q^{fb} so that the maximal joint surplus in a given period t equals Q^{fb} when $d_t = 1$; there are unique levels of work and search effort, e^{fb} and s^{fb}, so that the agent maximizes the joint surplus in period t, $Q_t = Q^{fb}$, if in all periods $\tau \geq t$ he chooses work effort $e_\tau = e^{fb}$, search effort $s_\tau = s^{fb}$, and $d_\tau = 1$ if and only if $r_\tau \leq Q^{fb}$.

4.2 Main Result

We examine the optimal incentive contract and the extent to which it implements the first-best outcome. First, we state its properties and then explain them step-by-step. By Lemma 1, we can focus on contracts in which the wage w_t and bonus b_t only depend on the current outside option value r_t. Thus, we can denote an optimal sequence of contract offers on the equilibrium path by $\{(w_t^*(r_t), b_t^*(r_t))\}_{t=0}^{\infty}$. Let $Q_t^*(r_t)$ be the joint surplus created under an optimal incentive contract at date t when $d_t = 1$ and the outside option value is r_t; and by $U_t^*(r_t)$ the corresponding payoff for the agent.

Proposition 1. *An optimal incentive contract has the following properties. The joint surplus is stationary and independent of the outside option,* $Q_t^*(r_t) = Q^*$*. In all periods t in which the agent works for the principal (i) joint surplus is not maximized,* $Q^* < Q^{fb}$*, (ii) the bonus is excessive,* $b_t^*(r_t) > \Delta y$*, and (iii) the agent may receive a rent in the sense that for some realizations of* r_t *we have* $U_t^*(r_t) > r_t$*.*

Inefficiency. The optimal incentive contract does not realize all gains from trade. The reason for this is not that the principal cannot provide sufficient effort incentives. In principle, she could offer a bonus in each period that makes the agent the residual claimant of the project, $b_t = \Delta y$, and extract the agent's gains through a negative wage. Indeed, this would be an optimal contract in the model when on-the-job search is costless or impossible.

Instead, the problem is that the agent's search incentives in any equilibrium are too large. To see this, note that finding an outside option can lead to two different outcomes. First, if the outside option value is (weakly) smaller than the joint surplus, $r_t \leq Q^*$, the agent continues to work for the principal.[13] The principal then offers a contract that matches the agent's outside option. Therefore, the agent may benefit from getting an outside offer even though he will not accept it. Second, if the outside option value is larger than the joint surplus in the project, $r_t > Q^*$, the agent rejects the principal's offer. It is then no longer profitable for the principal to keep the agent in the project since he would require an (expected) payment that exceeds the joint surplus from the project.

[13] For the argument in this and the next paragraph, it is not essential whether $Q^* < Q^{fb}$ or $Q^* = Q^{fb}$.

Search that is motivated by the first type of outside options – those that are useful for the agent because they improve his bargaining position – is pure rent-seeking. It causes search effort costs and drives up the costs of work effort, but only redistributes payoffs from the principal to the agent. Hence, it reduces the joint surplus. In contrast, search that is motivated by the second type of outside options – those that the agent actually chooses – increases the joint surplus. The principal could eliminate the rent-seeking motive in a given period t by promising the full surplus Q^* in period $t+1$ to the agent. The agent would then only search for outside options with values above Q^* so that his search effort s_t would be efficient. However, this promise is not credible. In period $t+1$, the principal would earn a zero payoff, regardless of the agent's outside option. As we show next, she then would like to renege on her promise and offer a different contract. Therefore, the agent invests too much into search under the optimal contract so that the joint surplus is below the first-best level.

Excessive Bonus. Next, we explain why the optimal contract features an excessive bonus. The principal faces an effort substitution problem in a multitask environment. The agent's excessive on-the-job search creates costs that lower the value of the relationship. To reduce the agent's search effort in period t, the principal can increase the bonus b_t, which makes work effort more attractive for the agent relative to search effort. All else being equal, work effort increases in the bonus, while search effort decreases in the bonus, $\frac{de_t}{db_t} > 0$ and $\frac{ds_t}{db_t} < 0$; see the appendix for details. As long as $b_t \leq \Delta y$, increasing the bonus also increases the joint surplus so that this measure is strictly beneficial for the principal. If $b_t > \Delta y$, further increasing the bonus creates benefits for the principal through lower search effort, but also costs since the principal has to compensate the agent for the increase in effort costs and this increase exceeds the corresponding raise in expected output. At $b_t = \Delta y$, increasing the bonus has first-order gains, but, by an envelope argument, only second-order costs. Thus, the optimal bonus strictly exceeds Δy. We can illustrate this argument formally. Under the optimal contract, the principal chooses w_t and b_t to maximize

$$V_t = (1-\delta)(\underline{y} - w_t + f(e_t)(\Delta y - b_t)) + \delta(1-p(s_t))(Q_{t+1} - U_{t+1}(0)) \\ + \delta p(s_t)G(Q_{t+1})(Q_{t+1} - \mathbb{E}_G[U_{t+1}(r_{r+1}) \mid r_{r+1} \leq Q_{t+1}]), \quad (8)$$

subject to the agent's incentive constraints in (6), (7), and the constraint that the agent earns his equilibrium level of utility, i.e.,

$$\max_{e_t,s_t} (1-\delta)(w_t + f(e_t)b_t - c(e_t, s_t)) + \delta(1-p(s_t))U_{t+1}(0)$$
$$+ \delta p(s_t)G(Q_{t+1})\mathbb{E}_G[U_{t+1}(r_{r+1}) \mid r_{r+1} \leq Q_{t+1}]$$
$$+ \delta p(s_t)(1-G(Q_{t+1}))\mathbb{E}_G[r_{r+1} \mid r_{r+1} > Q_{t+1}] \geq U_t(r_t). \tag{9}$$

From the last constraint we get that the principal reduces the fixed wage with the bonus at the rate $\frac{dw_t}{db_t} = -f(e_t)$ so that the first-order condition for an optimal bonus is given by

$$\frac{\partial V_t}{\partial b_t} = (1-\delta)f'(e_t)(\Delta y - b_t)\frac{de_t}{db_t} - \delta p'(s_t)(Q_{t+1} - U_{t+1}(0))\frac{ds_t}{db_t}$$
$$+ \delta p'(s_t)G(Q_{t+1})(Q_{t+1} - \mathbb{E}_G[U_{t+1}(r_{r+1}) \mid r_{r+1} \leq Q_{t+1}])\frac{ds_t}{db_t} = 0. \tag{10}$$

Since the principal cannot credibly promise the full joint surplus to the agent, we have

$$Q_{t+1} - U_{t+1}(0) > G(Q_{t+1})(Q_{t+1} - \mathbb{E}_G[U_{t+1}(r_{r+1}) \mid r_{r+1} \leq Q_{t+1}]). \tag{11}$$

Recall that $\frac{de_t}{db_t} > 0$ and $\frac{ds_t}{db_t} < 0$. At $b_t = \Delta y$, the principal's profit therefore strictly increases in the bonus; see the first-order condition in (10). Hence, the optimal bonus must be excessive.

Before we study the last feature of the optimal incentive contract, we introduce a special type of contract, the "no-rent contract" σ^{nr} (henceforth *nr*-contract). Under this contract, the principal offers in each period t a wage w_t that extracts all rents from the agent. Given the agent's expectations, $U_t = r_t$ for all t, the *nr*-contract specifies a bonus that maximizes the principal's expected payoff. This bonus is stationary (and thus also the joint surplus under the *nr*-contract). Denote by $w^{nr}(r_t)$ the wage, by b^{nr} the bonus, and by Q^{nr} the joint surplus under the *nr*-contract. If in some period t the agent accepts the *nr*-contract, the principal's total payoff is $Q^{nr} - r_t$, while the agent's total payoff is r_t. Since the *nr*-contract only uses the bonus to provide incentives and the bonus is contractible, the *nr*-contract is an equilibrium contract. Given that the agent expects that he will always only get the value of his outside option, there is no reason for the principal to pay more than the wage $w^{nr}(r_t)$ or to offer a different bonus. Hence, the agent's expectations are self-fulfilling.

Payoff Promises. The principal can pay an excessive bonus to keep the agent busy and distract him from on-the-job search. This measure is costly for the principal since she has to compensate the agent for the costs of excessive effort. However, principal and agent interact repeatedly so that there exists an alternative measure to reduce search incentives. Recall that under the *nr*-contract the agent's total payoff is perfectly correlated with his outside option value. The agent therefore has substantial incentives to exert search effort. One way to further curb search incentives would be to promise the agent a certain positive payoff in case his outside option value is zero or very small. The agent then has fewer incentives to avoid the state in which he has no outside offer.

Formally, the promise of a payoff of $\overline{u} < Q^{fb}$ in case of $r_{t+1} \leq \overline{u}$ changes search incentives as follows. Under the nr-contract the first-order condition that characterizes search in period t is given by

$$\frac{\delta}{1-\delta} p'(s_t) \int_0^{\infty} r_{t+1} g(r_{t+1}) dr_{t+1} = c_s(e_t, s_t). \tag{12}$$

If the principal promises a payoff of at least $\overline{u}$, the first-order condition becomes

$$\frac{\delta}{1-\delta} p'(s_t) \left(\int_{\overline{u}}^{\infty} r_{t+1} g(r_{t+1}) dr_{t+1} - (1 - G(\overline{u}))\overline{u} \right) = c_s(e_t, s_t). \tag{13}$$

The two first-order conditions are identical for $\overline{u} = 0$ and for any $\overline{u} > 0$ the left-hand side of (13) is strictly smaller than that of (12). So for given work effort search incentives decrease in the payoff promise. Indeed, we can show that in equilibrium work effort always increases in $\overline{u}$, while search effort always decreases in $\overline{u}$, $\frac{\partial e_t}{\partial \overline{u}} > 0$ and $\frac{\partial s_t}{\partial \overline{u}} < 0$; see the appendix for details.

What promise is credible and optimal for the principal? We introduce a new type of contract, the "payoff floor contract." We show under what circumstances there exists a payoff floor contract that is an equilibrium contract and that is strictly better for the principal than the nr-contract; then we show that some payoff floor contract is also an optimal contract.

On the equilibrium path of a payoff floor contract $\sigma^{\overline{u}}$, the principal offers the same bonus b^{pf} in each period; in period $t \geq 1$, the wage w_t is set so that $U_t(r_t) = \overline{u}$ if $r_t \leq \overline{u}$, and $U_t(r_t) = r_t$ if $r_t > \overline{u}$; in period $t = 0$ the wage is set so that $U_0 = 0$. Thus, in the initial period, the principal does not offer any rent to the agent, but she promises a positive rent in future periods when the outside option value is below $\overline{u}$. The nr-contract is a degenerate payoff floor contract with $\overline{u} = 0$ and $b^{pf} = b^{nr}$. A payoff floor contract with $\overline{u} = Q^{fb}$ and $b^{pf} = \Delta y$ would realize all gains from trade, but, as we noted above, it is not an equilibrium contract.

A payoff floor $\overline{u} > 0$ is credible only if it pays off for the principal to keep her promise in a period t with $r_t = 0$. In such a period, the principal's gains from the payoff floor must outweigh the costs of leaving a total payoff of $\overline{u}$ (instead of zero) to the agent. We construct a payoff floor contract by specifying that continuation play reverses to the nr-contract if the principal reneges on her promise. Intuitively, this means that if the principal does not keep her promise, the agent expects that she will extract all rents from him in all future periods.

We examine under what circumstances a payoff floor contract $\sigma^{\overline{u}}$ with $\overline{u} > 0$ is an equilibrium contract by using a perturbation argument. Start with the nr-contract and slightly reduce the bonus from b^{nr} to some value $b^{pf} < b^{nr}$. Then choose $\overline{u}$ so that search incentives remain constant, i.e., it is optimal for the agent to choose s^{nr} provided that the bonus is b^{pf} in each period and the principal always sets wages in $t \geq 1$ so that $U_t(r_t) = \overline{u}$ if $r_t \leq \overline{u}$, and $U_t(r_t) = r_t$ if $r_t > \overline{u}$. By construction, the agent then exerts less work effort than under the nr-contract; see the first-order condition (6). Since work effort under the nr-contract is excessive, this implies that the joint surplus increases. In other words, to reduce search incentives efficiently the principal replaces a fraction of the excessive bonus by the promise of not fully exploiting the agent if his outside option value is zero or very small (note, however, that the optimal contract may not leave search incentives constant at s^{nr}).

Observe that in period 0 and in a period t with $d_t = 1$ and $r_t > \overline{u}$, contract $\sigma^{\overline{u}}$ is strictly better for the principal than the nr-contract. The joint surplus Q_t is larger under $\sigma^{\overline{u}}$, but the agent's total payoff $U_t(r_t)$ is just the same as under the nr-contract. In a period t with $b_t = 1$ and $r_t < \overline{u}$, the payoff floor creates first-order costs for the principal. Whether current and future (expected) gains from the payoff floor exceed the costs of keeping the promise, depends on the production and cost functions as well as on the discount factor. We can show that there exists a payoff floor contract $\sigma^{\overline{u}}$ with $\overline{u} > 0$ that is an equilibrium contract if

$$(1-\delta)\left.\frac{de_t}{db_t}\right|_{s_t=s^{nr}} f'(e_t)(\Delta y - b_t) - \frac{d\overline{u}}{db_t} + \frac{\delta[(1-p(s^{nr})) + p(s^{nr})G(Q^{nr})]}{1-\delta[(1-p(s^{nr})) + p(s^{nr})G(Q^{nr})]}$$

$$\times\left((1-\delta)\left.\frac{de_t}{db_t}\right|_{s_t=s^{nr}} f'(e_t)(\Delta y - b_t) - (1-p(s^{nr}))\frac{d\overline{u}}{db_t}\right) < 0, \tag{14}$$

at σ^{nr} (we derive this term in the proof of Proposition 1). The first term in (14) captures the gains through the reduction in excessive effort; the second term is the payoff floor that is needed to keep search effort constant when the bonus is reduced;[14] the third term scales up the value of future gains by the discount factor and the continuation probability of the relationship; and the fourth term (in brackets) captures future gains from the payoff floor. Note that the fourth term is identical to the first two terms except that the costs of the payoff floor are weighted by their probability of occurrence. The inequality is satisfied if $p(s^{nr})$ is large enough so that the principal has to pay the payoff floor infrequently and $\delta G(Q^{nr})$ is sufficiently close to one so that the principal values the future benefits of the payoff floor (and in expectation the relationship endures for many periods); $G(Q^{nr})$ is close to one if most outside options have small values relative to the project payoffs $\underline{y}, \overline{y}$. Thus, there can exist a non-empty, closed and bounded set $\Lambda \in \mathbb{R}^+ \times \mathbb{R}^+$ of bonus and payoff floor combinations $(b^{pf}, \overline{u})$ that define payoff floor equilibrium contracts. By continuity, there then exists a combination $(b^{pf^*}, \overline{u}^*) \in \Lambda$ with positive $\overline{u}^*$ that defines an optimal payoff floor contract.

In our setup, there always exists a payoff floor contract that is also an optimal contract. Suppose that the optimal contract promises the agent payoffs above his reservation value in

[14] We have $\frac{ds_t}{d\overline{u}} d\overline{u} = -\frac{ds_t}{db_t} db_t$ so that $\frac{d\overline{u}}{db_t} = -\frac{ds_t}{db_t}\left(\frac{ds_t}{d\overline{u}}\right)^{-1}$.

some periods. If the contract is not a payoff floor contract then for some realizations of the outside option value, the agent receives a payoff that lies above his outside option value *and* above his payoff when his outside option has zero value, $U_t(r_t) > \max\{U_t(0), r_t\}$. Such a contract cannot be optimal because it generates search incentives that are too strong (i.e., by rearranging incentives, we can reduce the excessive work effort while keeping the agent's search effort constant). We therefore get the following result.[15]

Lemma 2. *There is a payoff floor contract with constant (excessive) bonus b^{pf^*} and payoff floor $\overline{u}^* \geq 0$ which is an optimal incentive contract.*

We can now precisely describe how the agent's costly search for better job opportunities changes the optimal incentive contract. Let us consider two benchmark cases: first, where search is impossible or prohibitively costly (e.g., there are no positions for the agent in other firms) and second, where search is costless (e.g., there are many positions for the agent in firms that are close nearby).[16] In the former case, the optimal contract implements the first-best allocation by paying the agent the full marginal surplus, $b_t = \Delta y$ in each period t, and extracting all rents from the agent through the wage. Similarly, in the latter case, the optimal contract implements the first-best by offering $b_t = \Delta y$ in each period t and a wage that makes the agent indifferent between his current outside offer and the continuation of the relationship. In contrast, when search is costly, the optimal incentive contract does not implement the first-best allocation. The principal pays an excessive bonus $b_t > \Delta y$ in each period t; and, depending on the parameters, she may also pay efficiency wages to the agent in the sense that the agent in some periods has a total payoff that exceeds his reservation value.

5 Extensions

To simplify the exposition of the model, we made a number of assumptions. In this section, we consider several extensions in which we relax some of them. In Subsection 5.1, we examine the baseline model with discretionary bonus payments. In Subsection 5.2, we assume that the agent is protected by limited liability. Finally, in Subsection 5.3, we discuss what happens to the optimal contract if search effort increases the chance of finding an outside option in several subsequent periods.

[15]This result may not hold when search effort also changes the distribution over outside option values G.

[16]Formally, prohibitively costly search means that $c(e_t, s_t) = \infty$ for all $s_t > 0$; costless search means that $c(e_t, s_t) = c(e_t, 0)$ for all $s_t > 0$.

5.1 Non-Contractible Bonus

In our baseline model, we assumed that the output is verifiable and the bonus enforceable. However, in many jobs, performance is difficult to objectively evaluate or verify to third parties. The relational contracts literature therefore analyzes optimal incentives when performance measures are non-verifiable. In the following, we show that our main result holds if we drop the assumption of a contractible bonus.

We update the setting from Section 3 so that the bonus is discretionary. In a contract offer (w_t, b_t), the term b_t now represents a promised bonus. After the output y_t is realized, the principal pays the wage w_t and chooses the realized bonus, which can be equal to b_t or any other non-negative value.[17] Both principal and agent can condition their actions on past bonus promises and realized bonus payments. In this setting, a promised bonus provides work incentives in equilibrium only if it is self-enforcing, i.e., it must be in the principal's best interest to pay the promised bonus. This constraint typically limits the size of the bonus.

We first define a continuation equilibrium σ^z for the case that the principal reneges on her bonus promise. In this equilibrium, the agent believes that the principal will never pay a bonus. Additionally, he assumes that in each period the principal offers a wage that makes the agent indifferent between the principal's contract and his outside option. Given these beliefs, it is optimal for the agent to exert no work effort, $e_t = 0$, and to choose search effort s_t so that $\frac{\delta}{1-\delta} p'(s_t)\mathbb{E}_G[r_t] = c_s(0, s_t)$. Denote by s^z the optimal search effort and by Q^z the joint surplus in the relationship if in each period t the agent exerts effort $e_t = 0$, $s_t = s^z$ and $d_t = 1$ if and only if $r_t \leq Q^z$. Given the agent's strategy, it is then indeed optimal for the principal to never pay a bonus and to extract all rents from the agent.

Consider the nr-contract from the last section. In the current setting, it is no longer clear whether the principal can credibly promise to pay the bonus b^{nr} after a high output. Suppose that continuation play is given by σ^z if the principal reneges on her promise. It is then rational for her to pay the promised bonus b^{nr} if and only if

$$\begin{aligned} -(1-\delta)b^{nr} + \delta(1 - p(s^{nr}))Q^{nr} + \delta p(s^{nr})G(Q^{nr})(Q^{nr} - \mathbb{E}_G[r_{t+1} \mid r_{t+1} \leq Q^{nr}]) \\ \geq \delta(1 - p(s^{nr}))Q^z + \delta p(s^{nr})G(Q^z)(Q^z - \mathbb{E}_G[r_{t+1} \mid r_{t+1} \leq Q^z]). \end{aligned} \tag{15}$$

We can show that this inequality is satisfied if (i) the probability of continuation of the relationship $G(Q^{nr})$ is high, (ii) the discount factor δ is close to 1, and (iii) Δy is large relative to $\mathbb{E}_G[r_t]\frac{c_{es}(e^{nr}, s^{nr})}{c_{ss}(e^{nr}, s^{nr})}$ so that work effort is important for the joint surplus, while the bonus b^{nr} is not too large; see the Appendix for details. Thus, the nr-contract can be an equilibrium contract in the setting with non-contractible bonus.

[17] For convenience, we do not explicitly introduce notation for the realized bonus.

Suppose that the *nr*-contract is an equilibrium contract. In this case, it is straightforward to show that if (14) is satisfied, then a payoff floor contract exists that is an equilibrium contract and strictly better for the principal than the *nr*-contract. Under a payoff floor contract, the principal has to keep two promises, i.e., to always offer a wage that implements the payoff floor and to pay the bonus b^{pf} in case of high output. We can specify that if the principal reneges on any of these promises, then continuation play is given by σ^z. Both condition (14) and condition (15) can be satisfied simultaneously so that a payoff floor contract can be the optimal contract under non-verifiable output. In fact, the set of payoff floors $\overline{u}$ which can occur in equilibrium is larger in the current setting than in the baseline model since the continuation equilibrium σ^z is strictly worse for the principal than σ^{nr}. We therefore have a relational contract model in which the optimal incentive contract may exhibit both excessive bonuses and efficiency wages.

Proposition 2. *In the model with non-contractible bonus, the nr-contract can be an equilibrium contract. An optimal incentive contract then has the following properties. The joint surplus is stationary and independent of the outside option,* $Q_t^*(r_t) = Q^*$*. In all periods t in which the agent works for the principal (i) joint surplus is not maximized,* $Q^* < Q^{fb}$*, (ii) the bonus is excessive,* $b_t^*(r_t) > \Delta y$*, and (iii) the agent may receive a rent in the sense that for some realizations of* r_t *we have* $U_t^*(r_t) > r_t$*.*

5.2 Limited Liability

The assumption of unlimited liability ensures that the principal can increase the bonus while keeping the agent's payoff constant by lowering the wage. In each period t, the optimal incentive contract therefore features an excessive bonus $b_t > \Delta y$ and the wage w_t is negative if the outside option value r_t is close enough to zero. However, in many jobs, negative wages are not feasible, for example, because of minimum wage requirements or because employees lack the financial resources to cover negative wages (e.g., franchising fees). In this subsection, we analyze our baseline model under the assumption of limited liability. We show that the optimal contract then may still pay excessive bonuses in some periods.

We consider the original setting under the assumption of limited liability. In each period t, the contract (w_t, b_t) must satisfy the constraints $w_t \geq \overline{w}$ and $w_t + b_t \geq \overline{w}$ for some $\overline{w} \in \mathbb{R}$. Note that all else being equal, the optimal contract in this setting is the same as in the baseline setting if $\overline{w}$ is sufficiently small. In the following, we therefore assume that $\overline{w} = 0$ so that limited liability changes the optimal contract. Lemma 1 remains valid in this setting. Any incentive effect that can be achieved through a variation in the agent's continuation value can also be generated through a raise in the bonus. Thus, we can again focus on equilibria in which the agent's total payoff in period t only depends on the realized outside option value r_t.

Recall the nr-contract from the baseline model. This contract cannot be an equilibrium contract since it pays a negative wage when the agent's outside option value is zero, $w^{nr}(0) < 0$. Thus, the bonus of the optimal contract – and hence the joint surplus – may vary with the agent's outside option value. Let σ^{ll} be an optimal contract and denote by $w_t^{ll}(r_t)$, $b_t^{ll}(r_t)$ the wage and bonus that are paid on the equilibrium path in period t when the agent's outside option is r_t. Let $U_t^{ll}(r_t)$ be the agent's corresponding total utility, and $Q_t^{ll}(r_t)$ the joint surplus in period t. Let Q_t^{max} be the highest value of r_t in period t so that the agent chooses the principal's contract (we assume that in case of indifference he works for the principal). Then the wage $w_t^{ll}(r_t)$ and the bonus $b_t^{ll}(r_t)$ are chosen to maximize

$$V_t^{ll}(r_t) = (1-\delta)(\underline{y} - w_t + f(e_t)(\Delta y - b_t)) + \delta(1-p(s_t))(Q_{t+1}^{ll}(0) - U_{t+1}^{ll}(0)) \\ + \delta p(s_t) G(Q_{t+1}^{max}) \mathbb{E}_G[Q_{t+1}^{ll}(r_{t+1}) - U_{t+1}^{ll}(r_{r+1}) \mid r_{t+1} \leq Q_{t+1}^{max}], \tag{16}$$

subject to the agent's incentive constraints in (6), (7), the constraint that the agent gets his equilibrium level of utility,

$$\max_{e_t, s_t} \; (1-\delta)(w_t + f(e_t)b_t - c(e_t, s_t)) + \delta(1-p(s_t))U_{t+1}^{ll}(0) \\ + \delta p(s_t) G(Q_{t+1}^{max}) \mathbb{E}_G[U_{t+1}^{ll}(r_{r+1}) \mid r_{r+1} \leq Q_{t+1}^{max}] \\ + \delta p(s_t)(1 - G(Q_{t+1}^{max})) \mathbb{E}_G[r_{r+1} \mid r_{r+1} > Q_{t+1}^{max}] \geq U_t^{ll}(r_t), \tag{17}$$

and the limited liability constraints. With this, we can show two features of the optimal contract. First, for large outside option values the bonus must be excessive. Intuitively, this is the case because in order to match the agent's outside option the principal has to pay a positive wage w_t when r_t is large. The limited liability constraint is then no longer binding so that (by the same logic as in the baseline model) the optimal bonus exceeds Δy. Thus, the bonus under the optimal contract may be non-stationary. Second, for small values of the outside option r_t the agent must earn a rent in the sense that $U_t^{ll}(r_t) > r_t$. The optimal wage at small r_t is zero due to limited liability, but the optimal bonus is strictly positive. Since $U_t^{ll}(r_t) > 0$ for all t and r_t, the agent's participation constraint is not binding when r_t is small or zero. Thus, limited liability mechanically creates a payoff floor which reduces search incentives.[18]

Proposition 3. *In the model with limited liability, an optimal incentive contract has the following properties. If for given parameters the low output $\underline{y}$ is sufficiently large, then in all periods t in which the agent works for the principal (i) the bonus is excessive when the outside*

[18]It is a standard result that the agent earns a rent if he is risk neutral and protected by limited liability.

option value r_t is sufficiently large and (ii) the agent receives a rent, $U_t^*(r_t) > r_t$, when the outside option value r_t is sufficiently close to zero.

5.3 Dynamic Effects of Search Effort

In our baseline model, search in period t only has an effect on the probability of finding an outside offer in period $t+1$. This assumption may be somewhat strict since previous job search may result in personal relationships that are useful for future search efforts. Suppose now that search in period t increases the probability of finding an outside option in all periods $t+1, t+2, ..., t+T$. This probability is now given by a function $p_t(s_{t-1}, ..., s_{t-T})$. In general, previous search then affects search incentives in the current period so that the optimal contract may be non-stationary (and thus more cumbersome to characterize). However, we can show for a special case that our main result holds when search has dynamic effects on the outside option value. Let the search technology be given by $p_t(s_{t-1}, ..., s_{t-T}) = p' \sum_{\tau=1}^{T} \alpha_\tau s_{t-\tau}$ for some value $p' > 0$ and $(\alpha_1, ..., \alpha_T)$.[19] Suppose that in equilibrium wage and bonus in period t only depend on the outside option value r_t. Then the agent's first-order condition for optimal search effort in period t is

$$\frac{1}{1-\delta} p' \sum_{\tau=1}^{T} \alpha_\tau \sum_{\tau=1}^{T} \delta^\tau (\mathbb{E}_G[U_{t+\tau}(r_{t+\tau})] - U_{t+\tau}(0)) = c_s(e_t, s_t). \tag{18}$$

The first-order condition for optimal work effort is again given by (6). Thus, both work and search effort in period t are independent of previous search efforts. Observe that we have $\mathbb{E}_G[U_t(r_t)] = r_t$ in any equilibrium in which the principal extracts all rents from the agent in all periods. In such an equilibrium, the search effort is again implicitly defined by (18). Thus, the nr-contract σ^{nr} is an equilibrium contract, and it exhibits the same features as in the baseline model. The optimal payoff floor contract $\sigma^{\bar{u}^*}$ is also an equilibrium contract since we can specify that continuation play is given by σ^{nr} if the principal fails to offer the wage/bonus combination prescribed by $\sigma^{\bar{u}^*}$. We therefore obtain the same results as in the baseline model.

6 Conclusion

In this paper, we investigated how the shape of the optimal incentive contract is affected when workers can generate competition for their services though on-the-job search. The importance of job-to-job transitions in today's labor markets suggests that on-the-job search is a significant issue for many employers. We showed that costly on-the-job search affects the incentive prob-

[19] We assume that p' is small enough such that optimal search effort is given by an interior solution.

lem inside organizations in meaningful ways. It creates value if its only motivation is a better worker-firm match. However, the agent can use outside offers from less productive relationships to negotiate a better deal with his principal. This rent-seeking motive causes the agent to invest too many resources in on-the-job search. The optimal incentive contract reduces this rent-seeking by offering excessive performance pay and potentially offering efficiency wages. The latter may come in the form of payoff floors: The principal does not extract all rents from the agent if his current outside option value is very small. Both measures reduce the agent's search incentives (but they are not perfect substitutes). Nevertheless, there is rent-seeking under the optimal contract. Therefore, on-the-job search creates agency costs.

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Appendix

Proof of Lemma 1. Let $\sigma = (\sigma_P, \sigma_A)$ be given and assume that history h_t^0 is reached upon which the principal offers the contract (w_t, b_t). Denote by $U_{t+1}(y_t, r_{t+1})$ the agent's continuation value in period $t+1$ if the outcome in period t was y_t and his outside option value in period $t+1$ is r_{t+1}. The agent's effort choices (e_t, s_t) solve

$$\max_{e_t,s_t} (1-\delta)(f(e_t)b_t - c(e_t,s_t)) + \delta(1-p(s_t))[f(e_t)U_{t+1}(\overline{y},0) + (1-f(e_t))U_{t+1}(\underline{y},0)] \\ + \delta p(s_t)\mathbb{E}_G[f(e_t)U_{t+1}(\overline{y},r_{t+1}) + (1-f(e_t))U_{t+1}(\underline{y},r_{t+1})]. \quad (19)$$

Define

$$\hat{b}_t = \frac{\delta}{1-\delta}(1-p(s_t))[U_{t+1}(\overline{y},0) - U_{t+1}(\underline{y},0)] + \frac{\delta}{1-\delta}p(s_t)\mathbb{E}_G[U_{t+1}(\overline{y},r_{t+1}) - U_{t+1}(\underline{y},r_{t+1})]. \quad (20)$$

We can rewrite the problem in (19) as

$$\max_{e_t,s_t} (1-\delta)(f(e_t)(b_t+\hat{b}_t) - c(e_t,s_t)) + \delta(1-p(s_t))U_{t+1}(\underline{y},0) + \delta p(s_t)\mathbb{E}_G[U_{t+1}(\underline{y},r_{t+1})]. \quad (21)$$

Consider the alternative profile $\hat{\sigma} = (\hat{\sigma}_P, \hat{\sigma}_A)$ where $\hat{\sigma}_P$ is identical to σ_P except that after history h_t^0 the principal offers contract $(w_t, b_t + \hat{b}_t)$ and her continuation play after any outcome y_t equals that under σ_P when $y_t = \underline{y}$; $\hat{\sigma}_A$ is identical to σ_A except that the agent accepts the contract after history $h_t^1 = (h_t^0, w_t, b_t + \hat{b}_t)$ and exerts the same effort as on the equilibrium path of the original equilibrium; his continuation play after any outcome y_t equals that under σ_A when $y_t = \underline{y}$. By construction, $\hat{\sigma}$ is an equilibrium and payoff equivalent to σ. The result then follows from applying this argument to all histories $h_t^0 \in H^0$. □

Marginal effects of the bonus b_t and payoff floor $\overline{u}$ on period-t efforts. Suppose that $d_t = 1$. Then the agent's problem is

$$\max_{e_t,s_t} (1-\delta)(w_t + f(e_t)b_t - c(e_t,s_t)) + \delta(1-p(s_t))\overline{u} \\ + \delta p(s_t)G(\overline{u})\overline{u} + \delta p(s_t)(1-G(\overline{u}))\mathbb{E}_G[r_{t+1} \mid r_{t+1} \geq \overline{u}]. \quad (22)$$

The agent's optimal choices are thus characterized by the first-order conditions

$$f'(e_t)b_t - c_e(e_t,s_t) = 0, \quad (23)$$

$$\frac{\delta}{1-\delta}p'(s_t)\left(-\overline{u} + G(\overline{u})\overline{u} + (1-G(\overline{u}))\mathbb{E}_G[r_{t+1} \mid r_{t+1} \geq \overline{u}]\right) - c_s(e_t,s_t) = 0. \quad (24)$$

Using the implicit function theorem we then get

$$\frac{de_t}{d\overline{u}} = \frac{-\frac{\delta}{1-\delta}p'(s_t)(1-G(\overline{u}))c_{es}(e_t,s_t)}{c_{ss}(e_t,s_t)(f''(e_t)b_t - c_{ee}(e_t,s_t)) + c_{es}(e_t,s_t)^2} > 0, \tag{25}$$

$$\frac{ds_t}{d\overline{u}} = \frac{-\frac{\delta}{1-\delta}p'(s_t)(1-G(\overline{u}))(f''(e_t)b_t - c_{ee}(e_t,s_t))}{c_{ss}(e_t,s_t)(f''(e_t)b_t - c_{ee}(e_t,s_t)) + c_{es}(e_t,s_t)^2} < 0, \tag{26}$$

where the inequalities follow from the assumptions on the production and cost functions. With respect to the bonus b_t we get

$$\frac{de_t}{db_t} = \frac{-c_{ss}(e_t,s_t)f'(e_t)}{c_{ss}(e_t,s_t)(f''(e_t)b_t - c_{ee}(e_t,s_t)) + c_{es}(e_t,s_t)^2} > 0, \tag{27}$$

$$\frac{ds_t}{db_t} = \frac{c_{es}(e_t,s_t)f'(e_t)}{c_{ss}(e_t,s_t)(f''(e_t)b_t - c_{ee}(e_t,s_t)) + c_{es}(e_t,s_t)^2} < 0. \tag{28}$$

We use these terms throughout the paper. □

Proof of Proposition 1. By Lemma 1 we can focus on contracts in which the principal's and agent's total payoff at the beginning of period t only depends on r_t. The proof proceeds in several steps. **Step 1.** We characterize the nr-contract. Under this contract, the agent chooses in each period t efforts (e_t, s_t) to maximize

$$U_t = (1-\delta)(w_t + f(e_t)b_t - c(e_t,s_t)) + \delta p(s_t)\mathbb{E}_G[r_{t+1}]. \tag{29}$$

By the assumptions on the production and cost functions, the effort combination (e_t, s_t) that maximizes U_t for given b_t is unique. Consider a period t in which the agent works for the principal and his outside option value is r_t. Let $\hat{Q}_{t+1}(r_{t+1}) = Q_{t+1}(r_{t+1})$ be the joint surplus in period $t+1$ if the agent works for the principal this period and his outside option value is r_{t+1}, and $\hat{Q}_{t+1}(r_{t+1}) = r_{t+1}$ if the agent rejects the principal's contract in period $t+1$. The principal's total payoff in period t is $V_t(r_t) = Q_t(r_t) - r_t$, where

$$Q_t(r_t) = (1-\delta)(\underline{y} + f(e_t)\Delta y - c(e_t,s_t)) + \delta(1-p(s_t))\hat{Q}_{t+1}(0) + \delta p(s_t)\mathbb{E}_G[\hat{Q}_{t+1}(r_{t+1})]. \tag{30}$$

Note that $Q_t(r_t)$ does not directly depend on r_t (it may depend on r_t only through e_t and s_t). Let Q^{nr} be the highest possible joint surplus in period $t+1$ when the agent works for the principal. Let b^{nr} maximize $Q_t(r_t)$ when $\hat{Q}_{t+1}(r_{t+1}) = Q^{nr}$ if $r_{t+1} \leq Q^{nr}$, and $\hat{Q}_{t+1}(r_{t+1}) = r_{t+1}$ otherwise. By the assumptions on the production and cost functions, b^{nr} is uniquely defined. By the stationarity of our setup, we have $Q_t(r_t) = Q^{nr}$ if $b_t = b^{nr}$; and $b_\tau = b^{nr}$ also maximizes $Q_\tau(r_\tau)$ in each period $\tau > t$ with $d_\tau = 1$. Thus, the joint surplus under the nr-contract is $Q_t = Q^{nr}$ if $d_t = 1$. Under the nr-contract, the agent works for the principal and exerts effort (e^{nr}, s^{nr})

in period t if $r_t \leq Q^{nr}$, and quits the relationship if $r_t > Q^{nr}$. **Step 2.** As in the text, we can show that the nr-contract features excessive bonuses, $b^{nr} > \Delta y$. Note that in the maximization problem in (8) to (9) we have $Q_{t+1} = Q^{nr}$ and $U_{t+1}(r_{t+1}) = r_{t+1}$. **Step 3.** We show that the joint surplus under the optimal contract is stationary and independent of the outside option, $Q_t^*(r_t) = Q^*$. Consider any period t in which the agent works for the principal. Under the optimal contract wage w_t and bonus b_t are chosen to maximize $V_t(r_t) = Q_t(r_t) - U_t(r_t)$. Note that for given expectations about $U_{t+1}(r_{t+1})$ the only variable that influences $Q_t(r_t)$ is the bonus b_t. We can vary b_t while keeping $U_t(r_t)$ fixed by adjusting w_t. Thus, $Q_t(r_t)$ is independent of r_t under an optimal contract. Assume by contradiction that there are two periods, t and t', so that on the equilibrium path of an optimal contract it can happen that $d_t = d_{t'} = 1$ and $Q_t < Q_{t'}$. Then we can increase the principal's payoff by continuing play in period t like in period t' and choosing w_t so that the original payoff promises in $U_t(r_t)$ are fulfilled. Therefore, the joint surplus in the relationship under an optimal contract must be stationary. As in Step 2, we can show that this implies that the optimal contract features excessive bonuses, $b_t^*(r_t) > \Delta y$. **Step 4.** We show that the optimal contract does not maximize the joint surplus. Suppose by contradiction that it does. Then, in each period t with $d_t = 1$, we must have $Q_t^* = Q^{fb}$ and $d_t = 1$ if $r_t < Q^{fb}$ as well as $d_t = 0$ if $r_t > Q^{fb}$. When $d_t = 1$, this is only possible if the agent has first-best work effort incentives, $b_t^* = \Delta y$, and first-best search effort incentives,

$$\frac{\delta}{1-\delta} p'(s^{fb})(1 - G(Q^{fb}))(\mathbb{E}_G[r_{t+1} \mid r_{t+1} \geq Q^{fb}] - Q^{fb}) = c_s(e^{fb}, s^{fb}). \tag{31}$$

The latter is possible only if $U_t(r_t) = Q^{fb}$ whenever $d_t = 1$ and $r_t < Q^{fb}$. This implies that the agent receives the entire rent from the project. Promising the entire rent to the agent is not credible. For $r_{t+1} < Q^{nr}$ the principal would have an incentive to return to the nr-contract in period $t + 1$ as it generates a strictly positive payoff for her. **Step 5.** We derive a condition under which there exists a payoff floor contract that is an equilibrium contract and is strictly better for the principal than the nr-contract. Denote by $w^{nr}(r_t)$ the wage in period t under the nr-contract if the outside option value is r_t. Consider the following change in σ^{nr}. The principal reduces the bonus from b^{nr} to b^{pf} and then chooses $\overline{u}$ so that the agent's search effort remains constant, that is, the agent's optimal search effort is s^{nr} provided that $d_t = 1$, $b_t = b^{pf}$, and his continuation utility is $U_{t+1}(r_{t+1}) = r_{t+1}$ when $r_{t+1} \geq \overline{u}$ and $U_{t+1}(r_{t+1}) = \overline{u}$ otherwise. Denote by e^{pf} the agent's work effort given that his search effort is s^{nr} and the bonus is b^{pf}. By construction, we have $e^{pf} < e^{nr}$. Let $w^{pf}(r_t)$ be the wage that implements the payoff floor, i.e.,

$$\max_{e_t,s_t} (1-\delta)(w^{pf}(r_t) + f(e_t)b^{pf} - c(e_t,s_t)) + \delta(1-p(s_t))\overline{u} + \delta p(s_t)G(\overline{u})\overline{u} + \delta p(s_t)(1-G(\overline{u}))\mathbb{E}_G[r_{t+1} \mid r_{t+1} \geq \overline{u}] = \max\{r_t, \overline{u}\}. \quad (32)$$

We must have

$$w^{pf}(r_t) \leq w^{nr}(r_t) + \frac{1}{1-\delta}\max\{\overline{u}-r_t, 0\} + \int_0^{e^{pf}} f'(e)(b^{nr}-b^{pf})de + \int_{e^{pf}}^{e^{nr}} f'(e)b^{nr} - c_e(e, s^{nr})de. \quad (33)$$

Note that the third term on the right-hand side is an expected payment that the principal also pays under the original contract through the bonus and the marginal increase in the fourth term as b^{pf} is reduced starting from b^{nr} is zero. Given that $d_t = 1$, the difference in the principal's normalized *expected* period-t payoff between the new contract σ^{pf} and σ^{nr} is at least

$$(1-\delta)(f(e^{pf}) - f(e^{nr}))(\Delta y - b^{nr}) - (1-\delta)\int_{e^{pf}}^{e^{nr}} f'(e)b^{nr} - c_e(e, s^{nr})de - (1-p(s^{nr}))\overline{u} - p(s^{nr})\int_0^{\overline{u}} g(r)(\overline{u}-r)dr. \quad (34)$$

Thus, in a period t with $r_t = 0$, the principal's total expected payoff V_t is larger under σ^{pf} than under σ^{nr} if b^{pf} is close enough to b^{nr} and condition (14) holds at σ^{nr}. Denote the left-hand side of (14) by Γ. In the following, we derive an alternative expression for $f'(e_t)(\Delta y - b_t)$ in Γ. Note that b^{nr} and $w^{nr}(r_t)$ maximize

$$\begin{aligned} V_t(r_t) &= (1-\delta)(\underline{y} - w_t + f(e_t)(\Delta y - b_t)) \\ &\quad +\delta(1-p(s_t))Q_{t+1} + \delta p(s_t)G(Q_{t+1})(Q_{t+1} - \mathbb{E}_G[r_{r+1} \mid r_{r+1} \leq Q_{t+1}]), \end{aligned} \quad (35)$$

subject to the agent's incentive constraints in (6) and (7), as well as the participation constraint

$$\max_{e_t,s_t}(1-\delta)(w_t + f(e_t)b_t - c(e_t,s_t)) + \delta p(s_t)\mathbb{E}_G[r_{t+1}] \geq r_t. \quad (36)$$

Since the participation constraint must be binding, we have

$$(1-\delta)w_t = r_t + (1-\delta)c(e_t,s_t) - (1-\delta)f(e_t)b_t - \delta p(s_t)\mathbb{E}_G[r_{r+1}], \quad (37)$$

so that $\frac{dw_t}{db_t} = -f(e_t)$. Thus, b^{nr} is characterized by

$$\frac{\partial V_t}{\partial b_t} = (1-\delta)f'(e_t)(\Delta y - b_t)\frac{de_t}{db_t} - \delta p'(s_t)(1-G(Q_{t+1}))Q_{t+1}\frac{ds_t}{db_t}$$
$$- \delta p'(s_t)G(Q_{t+1})\mathbb{E}_G[r_{r+1} \mid r_{r+1} \leq Q_{t+1}]\frac{ds_t}{db_t} = 0 \tag{38}$$

at σ^{nr}. Thus, we must have

$$f'(e_t)(\Delta y - b_t) \geq \frac{\delta}{1-\delta}p'(s_t)G(Q_{t+1})\mathbb{E}_G[r_{r+1} \mid r_{r+1} \leq Q_{t+1}]\frac{ds_t}{db_t}\left(\frac{de_t}{db_t}\right)^{-1} \tag{39}$$

at σ^{nr}. Replacing $f'(e_t)(\Delta y - b_t)$ in Γ by the right-hand side of this inequality yields us the desired condition. **Step 6.** We show by example that the condition $\Gamma < 0$ in (14) can be satisfied and we examine the conditions under which this is the case. Assume

$$c(e_t, s_t) = \frac{1}{2}\beta_1 e_t^2 + \frac{1}{2}\beta_2 e_t^2 s_t^2 + \frac{1}{2}\beta_3 s_t^2, \tag{40}$$

$f(e_t) = f' e_t$, $p(s_t) = p' s_t$, and $\overline{e} = \overline{s} = 1$. Consider the term in squared brackets in (14),

$$(1-\delta)\left.\frac{de_t}{db_t}\right|_{s_t = s^{nr}} f'(e_t)(\Delta y - b_t) + (1 - p(s^{nr}))\frac{ds_t}{db_t}\left(\frac{ds_t}{d\overline{u}}\right)^{-1}. \tag{41}$$

By the inequality in (39), this term is strictly negative in the example if

$$\frac{\delta^2}{1-\delta}\frac{(p')^2}{c_{ss}(e_t, s_t)}G(Q^{nr})\mathbb{E}_G[r_{t+1} \mid r_{t+1} \leq Q^{nr}] > 1 - p(s^{nr}) \tag{42}$$

at σ^{nr}. We now show in several steps that the left-hand side of this inequality can be much larger than the right-hand side. First, consider the cost term $c_{ss}(e_t, s_t)$. It must be large enough so that search effort is given by an interior solution. From the first-order condition that characterizes optimal search, we get that

$$s_t \in \left(\frac{\delta}{1-\delta}\frac{1}{\beta_2+\beta_3}p'\mathbb{E}_G[r_{t+1}], \frac{\delta}{1-\delta}\frac{1}{\beta_3}p'\mathbb{E}_G[r_{t+1}]\right) \tag{43}$$

at σ^{nr}. To ensure that $s_t < \overline{s}$, we assume $\beta_3 \approx \frac{\delta}{1-\delta}p'\mathbb{E}_G[r_{t+1}]$. Next, consider the terms $G(Q^{nr})$ and $\mathbb{E}_G[r_{t+1} \mid r_{t+1} \leq Q^{nr}]$. Recall that

$$Q^{nr} = (1-\delta)[\underline{y} + f(e^{nr})\Delta y - c(e^{nr}, s^{nr})] + \delta(1 - p(s^{nr}))Q^{nr}$$
$$+ \delta p(s^{nr})G(Q^{nr})Q^{nr} + \delta p(s^{nr})(1 - G(Q^{nr}))\mathbb{E}_G[r_{t+1} \mid r_{t+1} \leq Q^{nr}]. \tag{44}$$

Since e^{nr} and s^{nr} do not depend on $\underline{y}$, Q^{nr} strictly increases in $\underline{y}$. Hence, we can find a distribution G that puts most probability mass on small realizations of r_t and $\underline{y}$ so that $G(Q^{nr})$ is close to 1 and $\mathbb{E}_G[r_{t+1} \mid r_{t+1} \leq Q^{nr}] \approx \mathbb{E}_G[r_{t+1}]$. Finally, consider the term $1 - p(s^{nr})$. Given that $\beta_3 \approx \frac{\delta}{1-\delta} p' \mathbb{E}_G[r_{t+1}]$, this term is close to zero if p' is close to 1 and β_2 is small relative to β_3. So we can find parameters such that at σ^{nr} the left-hand side of (42) is close to 1, the right-hand side is close to 0, $\delta \approx 1$, and $G(Q^{nr}) \approx 1$, so that overall $\Gamma < 0$. **Step 7.** We now can show that under the optimal contract the agent may receive a rent in the sense that for some realizations of r_t we have $U_t^*(r_t) > r_t$. Note that the best equilibrium contract under which the principal extracts all rents from the agent in all periods is the *nr*-contract. If condition (14) is satisfied, there exists a payoff floor contract $\sigma^{\overline{u}}$ with $\overline{u} > 0$ which is an equilibrium contract and that is strictly better for the principal than the *nr*-contract. Thus, the result follows from Step 5 and Step 6. □

Proof of Lemma 2. Assume by contradiction that there is an optimal incentive contract σ that is strictly better for the principal than the optimal payoff floor contract $\sigma^{\overline{u}^*}$. Then under σ there exists an interval $(r_L, r_H) \subset (0, \infty)$ and a period t such that $U_{t+1}(r_{t+1}) > \max\{U_{t+1}(0), r_{t+1}\}$ for all $r_{t+1} \in (r_L, r_H)$.[20] Select such an interval so that reducing $U_{t+1}(r_{t+1})$ to $\max\{U_{t+1}(0), r_{t+1}\}$ for all $r_{t+1} \in (r_L, r_H)$ would not violate the constraint that $U_{t+1}(r_{t+1})$ must be weakly increasing in r_{t+1}. Denote by $w_t(r_t)$ the agent's wage and by $b_t(r_t)$ the agent's bonus on the equilibrium path in period t under σ if his outside option is r_t. Denote by $e_t(r_t)$ and $s_t(r_t)$ the corresponding effort levels. By the arguments presented in the text, the bonus must be excessive, $b_t(r_t) > \Delta y$. To show that σ is not optimal for the principal, we construct an alternative contract $\hat{\sigma}$ that is strictly better for the principal. Define $w_{t+1}^{[1]}(r_{t+1}) = w_{t+1}(r_{t+1})$ for all $r_{t+1} \in [0, \infty] \setminus (r_L, r_H)$ and

$$w_{t+1}^{[1]}(r_{t+1}) = w_{t+1}(r_{t+1}) - \frac{1}{1-\delta}[U_{t+1}(r_{t+1}) - \max\{U_{t+1}(0), r_{t+1}\}] \tag{45}$$

for all $r_{t+1} \in (r_L, r_H)$. Next, define

$$w_t^{[1]}(r_t) = w_t(r_t) + p(s_t(r_t))\frac{\delta}{1-\delta}\int_{r_L}^{r_H}[U_{t+1}(r_{t+1}) - \max\{U_{t+1}(0), r_{t+1}\}]g(r_{t+1})dr_{t+1}. \tag{46}$$

Note that if we keep $e_t(r_t)$ and $s_t(r_t)$ fixed, the period-t payoff of each party remains the same when, on the equilibrium path of σ, we replace $w_t(r_t)$ by $w_t^{[1]}(r_t)$ and $w_{t+1}(r_{t+1})$ by $w_{t+1}^{[1]}(r_{t+1})$. In equilibrium, the agent's search effort is implicitly defined by

$$\frac{\delta}{1-\delta}p'(s_t(r_t))\int_0^{\infty} U_{t+1}(r_{t+1})g(r_{t+1})dr_{t+1} = c_s(e_t(r_t), s_t(r_t)). \tag{47}$$

[20]Here we use the statement from footnote 11.

Hence, under the new wages, search incentives in period t are smaller. Let $\sigma^{[1]}$ be identical to σ except that on the equilibrium path the wage in period t is given by $w_t^{[1]}(r_t)$, the wage in period $t+1$ is given by $w_{t+1}^{[1]}(r_{t+1})$, and the bonus $b_t^{[1]}(r_t)$ is chosen so that the agent's search effort in period t is the same as under σ when $d_t = 1$. By construction, we have $\Delta y < b_t^{[1]}(r_t) < b_t(r_t)$ for all r_t and therefore smaller work effort in period t under $\sigma^{[1]}$ than under σ when $d_t = 1$. Denote by $e_t^{[1]}(r_t)$ the search effort in period t under $\sigma^{[1]}$ when $d_t = 1$. To compensate the agent for the lower bonus, we choose

$$w_t^{[2]}(r_t) = w_t^{[1]}(r_t) + \int_0^{e^{[1]}(r_t)} f'(e)(b_t(r_t) - b_t^{[1]}(r_t))de + \int_{e^{[1]}(r_t)}^{e_t(r_t)} f'(e)b_t(r_t) - c_e(e, s_t(r_t))de. \tag{48}$$

Let $\sigma^{[2]}$ be identical to $\sigma^{[1]}$ except that the wage in period t on the equilibrium path is given by $w_t^{[2]}(r_t)$. The inequality $\Delta y < b_t^{[1]}(r_t) < b_t(r_t)$ implies that $\frac{\partial Q_t}{\partial e_t} < 0$ at all effort levels $e \in [e_t^{[1]}(r_t), e_t(r_t)]$. Hence, given that $d_t = 1$, we have for any r_t that

$$V_t^{[2]}(r_t) = Q_t^{[2]}(r_t) - U_t^{[2]}(r_t) = Q_t^{[2]}(r_t) - U_t(r_t) > Q_t(r_t) - U_t(r_t) = V_t(r_t), \tag{49}$$

which yields the contradiction. □

Proof of Proposition 2. We examine under what conditions the inequality in (15) holds. As in Step 5 of the proof of Proposition 1 we derive that

$$f'(e_t)(\Delta y - b^{nr}) = \frac{\delta}{1-\delta} p'(s_t)\left((1 - G(Q^{nr}))Q^{nr} + G(Q^{nr})\mathbb{E}_G[r_{r+1} \mid r_{r+1} \le Q^{nr}]\right) \frac{ds_t}{db_t}\left(\frac{de_t}{db_t}\right)^{-1} \tag{50}$$

at σ^{nr}. Consider the limit case with $\mathbb{E}_G[r_{r+1} \mid r_{r+1} \le Q^{nr}] = \mathbb{E}_G[r_{r+1}]$. In this case, we have

$$b^{nr} = \Delta y + \frac{1}{1-\delta}\frac{p'(s^{nr})}{f'(e^{nr})}\mathbb{E}_G[r_{r+1}]\frac{c_{es}(e^{nr}, s^{nr})}{c_{ss}(e^{nr}, s^{nr})}. \tag{51}$$

In the limit case, the bonus b^{nr} and wage $w^{nr}(r_t)$ maximize

$$V_t(r_t) = (1-\delta)(\underline{y} + f(e_t)(\Delta y - b_t) - w_t) + \delta[\underline{y} + f(e_t)\Delta y - c(e_t, s_t)] - \delta p(s_t)\mathbb{E}_G[r_{t+1}] \tag{52}$$

subject to the agent's incentive constraints in (6) and (7), as well as the participation constraint

$$\max_{e_t, s_t}(1-\delta)(w_t + f(e_t)b_t - c(e_t, s_t)) + \delta p(s_t)\mathbb{E}_G[r_{t+1}] \ge r_t. \tag{53}$$

The participation constraint implies

$$w_t = c(e_t, s_t) - f(e_t)b_t + \frac{1}{1-\delta}(r_t - \delta p(s_t)\mathbb{E}_G[r_{t+1}]). \tag{54}$$

We plug this back into (52) and see that b^{nr} maximizes $f(e_t)\Delta y - c(e_t, s_t)$ so that

$$f(e^{nr})\Delta y - c(e^{nr}, s^{nr}) > f(0)\Delta y - c(0, s^z) = -c(0, s^z). \tag{55}$$

In the limit case, condition (15) is implied by

$$-(1-\delta)b^{nr} + \delta Q^{nr} > \delta Q^z, \tag{56}$$

where $Q^{nr} = \underline{y} + f(e^{nr})\Delta y - c(e^{nr}, s^{nr})$ and $Q^z \leq \underline{y} + f(0)\Delta y - c(0, s^z)$. Hence, the inequality in (56) is satisfied if $(1-\delta)b^{nr}$ is sufficiently small, which by (51) is the case if δ is sufficiently close to 1 and $\mathbb{E}_G[r_{t+1}]\frac{c_{es}(e^{nr},s^{nr})}{c_{ss}(e^{nr},s^{nr})}$ is sufficiently small. In our example in (40), this latter condition holds if β_2 is sufficiently small relative to β_3. The first result in the proposition then follows from the fact that the principal's and the agent's payoff under the two contracts σ^{nr} and σ^z moves continuously in the distribution G and the search function p. The rest of the proposition can be shown as in the proof of Proposition 1 using the fact that both (14) and (56) can simultaneously be satisfied (observe that $c_{es}(e^{nr}, s^{nr})$ does not influence the sign of Γ). □

Proof of Proposition 3. The proof proceeds by steps. **Step 1.** We show that for large outside option values r_t the optimal contract features an excessive bonus, $b^{ll}(r_t) > \Delta y$. Note that Q_t^{max} strictly increases in $\underline{y}$. Consider the participation constraint in (17). The agent's total utility strictly increases in w_t and b_t. Suppose that $b_t^{ll}(r_t) \leq \Delta y$ for all $r_t \in [0, Q_t^{max}]$. If $\underline{y}$ is large enough, then for sufficiently large outside option values $r_t \leq Q_t^{max}$ we must have $w_t^{ll}(r_t) > \Delta y$. We then can offer a bonus of $\hat{b}_t^{ll}(r_t) = \Delta y$ and adjust $w_t^{ll}(r_t)$ to $\hat{w}_t^{ll}(r_t)$ so that the agent's total utility remains at $U_t^{ll}(r_t)$. Since $w_t^{ll}(r_t) > \Delta y$ we must have $\hat{w}_t^{ll}(r_t) > 0$. Thus, we can further increase $\hat{b}_t^{ll}(r_t)$ while keeping the agent's total utility constant. As discussed in Section 4, this change generates first-order gains, but only second-order costs for the principal. Thus, the optimal bonus must be excessive. **Step 2.** We show that the optimal contract features a payoff floor. Suppose that $r_t = 0$ and assume by contradiction that $U_t^{ll}(r_t) = 0$. Consider the participation constraint in (17). By limited liability, we have $U_{t+1}^{ll}(r_{t+1}) > 0$ for all r_{t+1} so that the participation is satisfied for all contracts (w_t, b_t) with $w_t \geq 0$ and $w_t + b_t \geq 0$. Thus, the optimal wage in period t is $w_t = 0$ and the maximization problem becomes

$$\begin{aligned}\max_{b_t \geq 0}\ & (1-\delta)(\underline{y} + f(e_t)(\Delta y - b_t)) + \delta(1 - p(s_t))(Q_{t+1}^{ll}(0) - U_{t+1}(0)) \\ & + \delta p(s_t)G(Q_{t+1}^{max})\mathbb{E}_G[Q_{t+1}^{ll}(r_{t+1}) - U_{t+1}^{ll}(r_{r+1}) \mid r_{t+1} \leq Q_{t+1}^{max}].\end{aligned} \tag{57}$$

Since in period t the principal can influence $Q_t^{ll}(r_t)$ only through the bonus $b_t(r_t)$ (provided that the agent's expectations are fixed), $Q_{t+1}^{ll}(r_{t+1}) - U_{t+1}^{ll}(r_{r+1})$ cannot be increasing in r_{r+1}.

Moreover, $Q^{ll}_{t+1}(r_{t+1}) - U^{ll}_{t+1}(r_{r+1})$ must be decreasing for some value of $r_{t+1} \leq Q^{max}_{t+1}$; otherwise, the principal's payoff would always be zero. The first-order condition for the optimal bonus in period t is

$$\begin{aligned}\frac{\partial V_t}{\partial b_t} &= (1-\delta)f'(e_t)(\Delta y - b_t)\frac{de_t}{db_t} - (1-\delta)f(e_t) - \delta p'(s_t)(Q^{ll}_{t+1}(0) - U^{ll}_{t+1}(0))\frac{ds_t}{db_t} \\ &\quad + \delta p'(s_t)\mathbb{E}_G[Q^{ll}_{t+1}(r_{t+1}) - U^{ll}_{t+1}(r_{r+1}) \mid r_{t+1} \leq Q^{max}_{t+1}]\frac{ds_t}{db_t} = 0. \qquad (58)\end{aligned}$$

At $b_t = 0$ we have $e_t = 0$ so that the left-hand side is strictly positive. Thus, the optimal bonus is positive. The result then follows from the free disposal assumption. □